



TECHNISCHE UNIVERSITÄT
BERGAKADEMIE FREIBERG

Die Ressourcenuniversität. Seit 1765.

Benchmark Problem for the Stability of a Retaining Wall



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Geotechnical Problems

Numerical analysis of geotechnical problems:

**Serviceability limit state (SLS):
Deformation based approach**

**Ultimate limit state analysis (ULS):
Strength based approach
EQU, HYD, UPL, GEO-2, STR, GEO-3**

Numerical analysis of geotechnical problems:

Geostatic (initial) stress state analysis

Sequence of (many) construction phases

Material fill, removal and replacement

**Potential change of material behaviour
during and between construction phases**

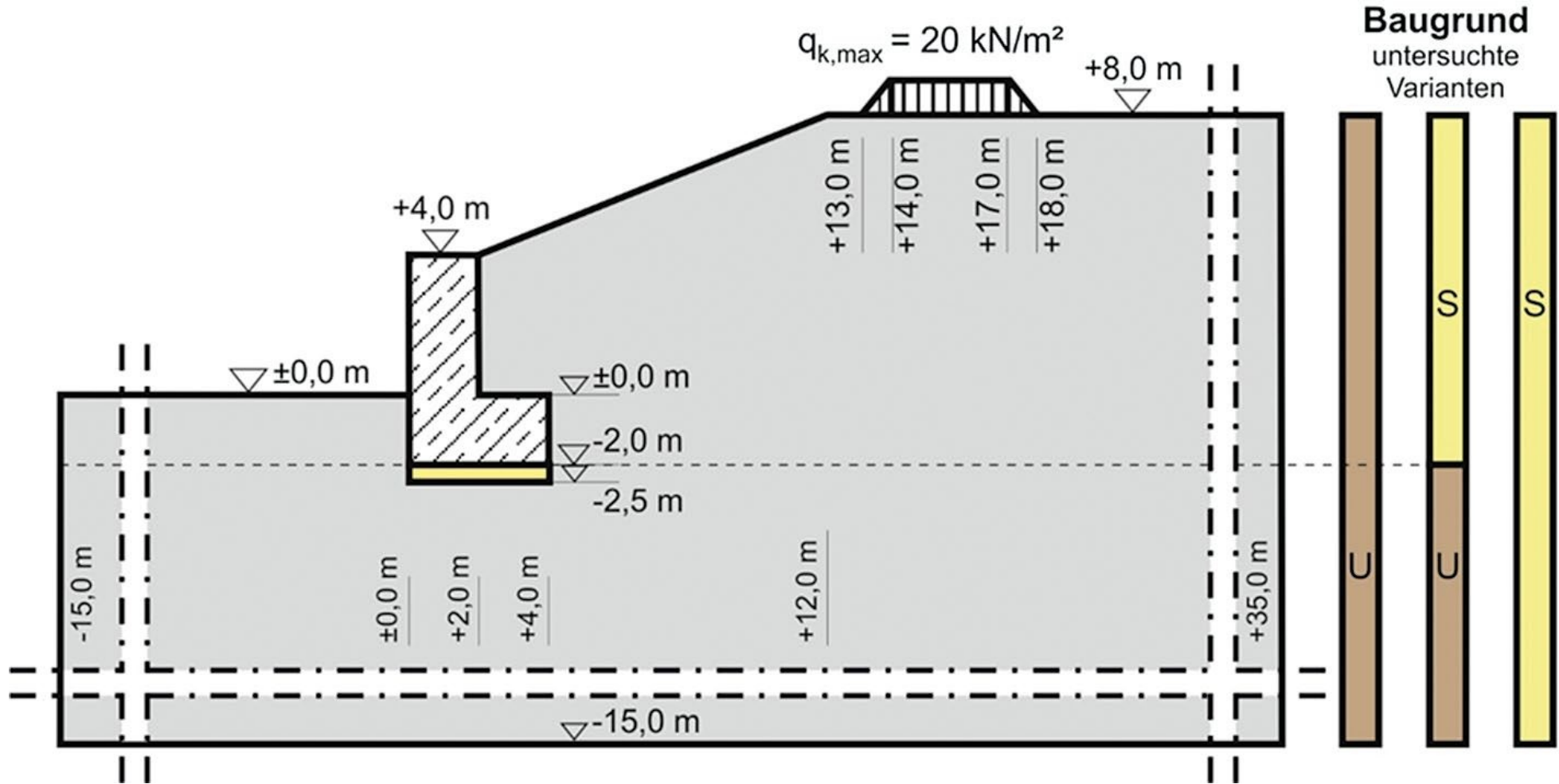
Numerical analysis of geotechnical problems:

**Special attention is required to
displacement, stress,
internal variable and material property
fields and their eventual
superposition or manipulation
between load phases**



Benchmark Problem of a Retaining Wall

Benchmark Problem for the Stability of a Retaining Wall



Benchmark Problem for the Stability of a Retaining Wall – Geometry and soil layering
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Benchmark problem – material properties:

Silt (U /Si/):

$$E = 40 \text{ [MN/m}^2\text{]}$$

$$\nu = 0.30 \text{ [1]}$$

$$\gamma = 20 \text{ [kN/m}^2\text{]}$$

$$\varphi_k = 20 \text{ [}^\circ\text{]}$$

$$c_k = 10.0 \text{ [kN/m}^2\text{]}$$

$$\psi = 0.0 \text{ [}^\circ\text{]}$$

$$\sigma_t = \mathbf{0.0} \text{ [kN/m}^2\text{]}$$

Sand (S /Sa/)

$$E = 40 \text{ [MN/m}^2\text{]}$$

$$\nu = 0.30 \text{ [1]}$$

$$\gamma = 20 \text{ [kN/m}^2\text{]}$$

$$\varphi_k = 35 \text{ [}^\circ\text{]}$$

$$c_k = 0.1 \text{ [kN/m}^2\text{]}$$

$$\psi = 5.0 \text{ [}^\circ\text{]}$$

$$\sigma_t = \mathbf{0.0} \text{ [kN/m}^2\text{]}$$

Concrete:

$$E = 40 \text{ [GN/m}^2\text{]}$$

$$\nu = 0.20 \text{ [1]}$$

$$\gamma = 25 \text{ [kN/m}^2\text{]}$$

Benchmark problem – configurations:

Sand basemat under retaining wall

Case SiSi: Silt below and Silt beyond foundation level

Case SiSa: Silt below and Sand beyond foundation level

Case SaSa: Sand below and Sand beyond foundation level

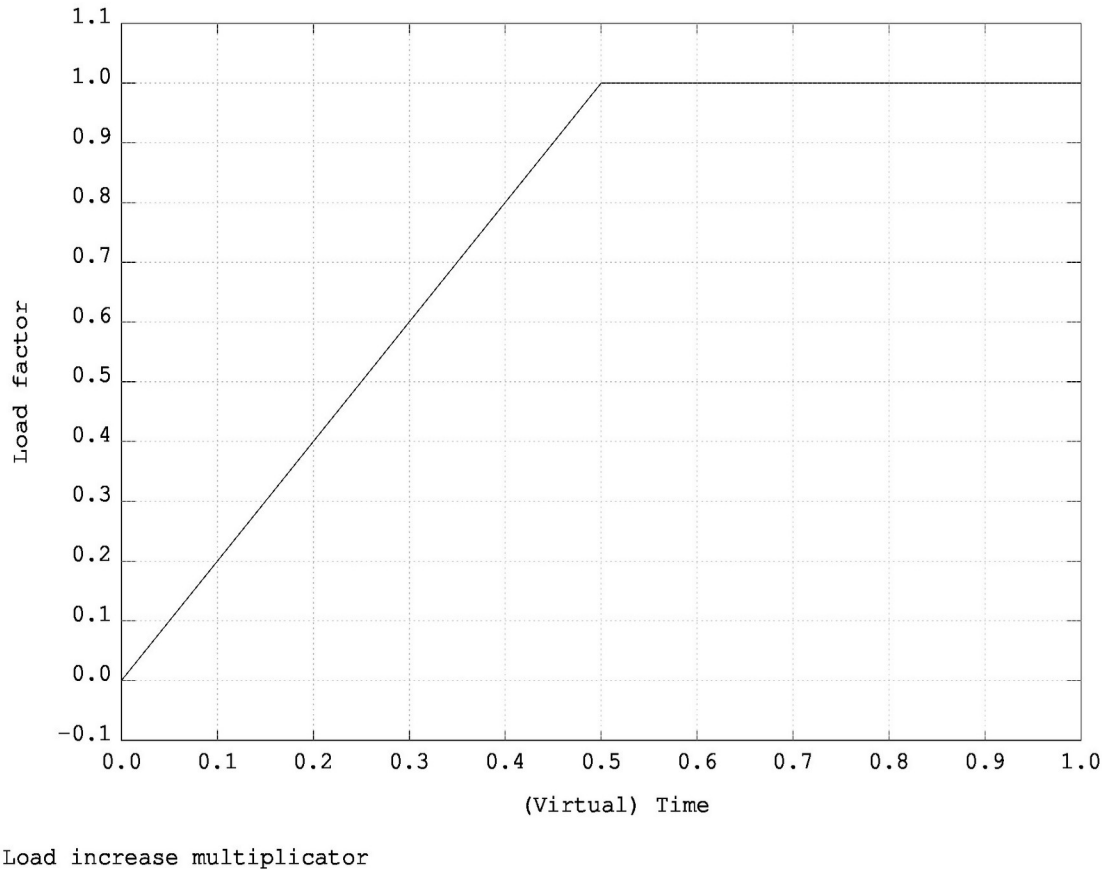
Benchmark problem – analysis phases:

- 1. Geostatic stress state in the basement (stress adjustment or gravity loading)**
- 2. Wall construction**
- 3. Layered backfill (simplified)**
- 4. Surface load (regarded as variable)**
- 5. General stability of the construction**

Benchmark problem – solution strategy:

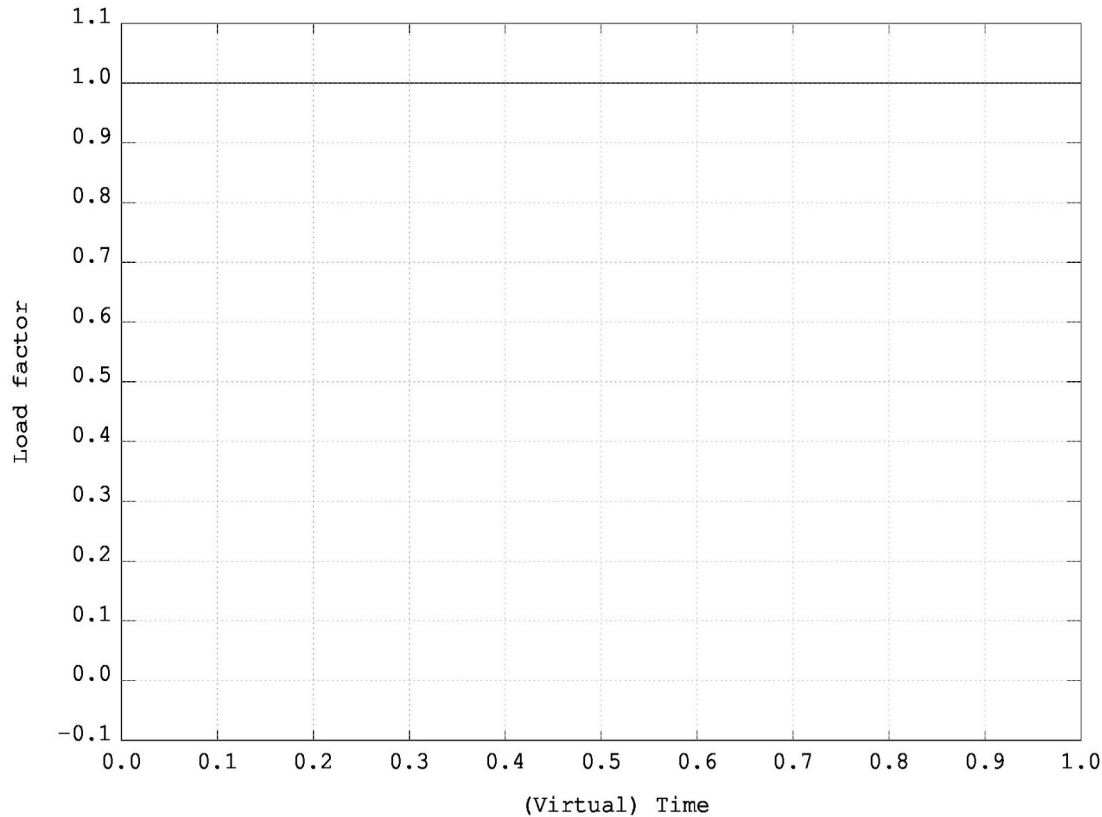
- 1. Treatment of the problem in plane strain**
- 2. Definition of soil and retaining wall clusters, surfaces and contours**
- 3. Definition of ALL external loads (self weight and charges)**
- 4. Implementation of the construction phases by stepwise imposition of loads**

Increasing load multiplier function with equilibrium iteration



Benchmark Problem for the Stability of a Retaining Wall – Increasing load
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Constant load multiplier function

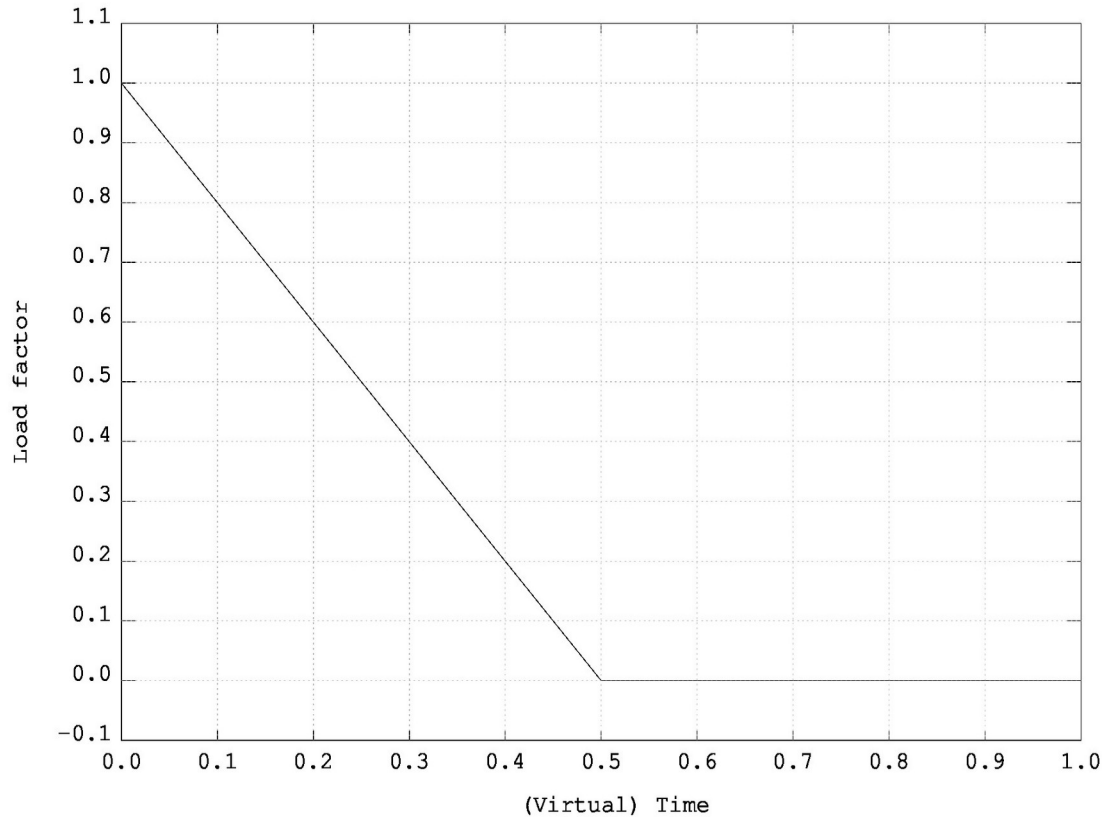


Load constant multiplier

Benchmark Problem for the Stability of a Retaining Wall – Constant load

DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

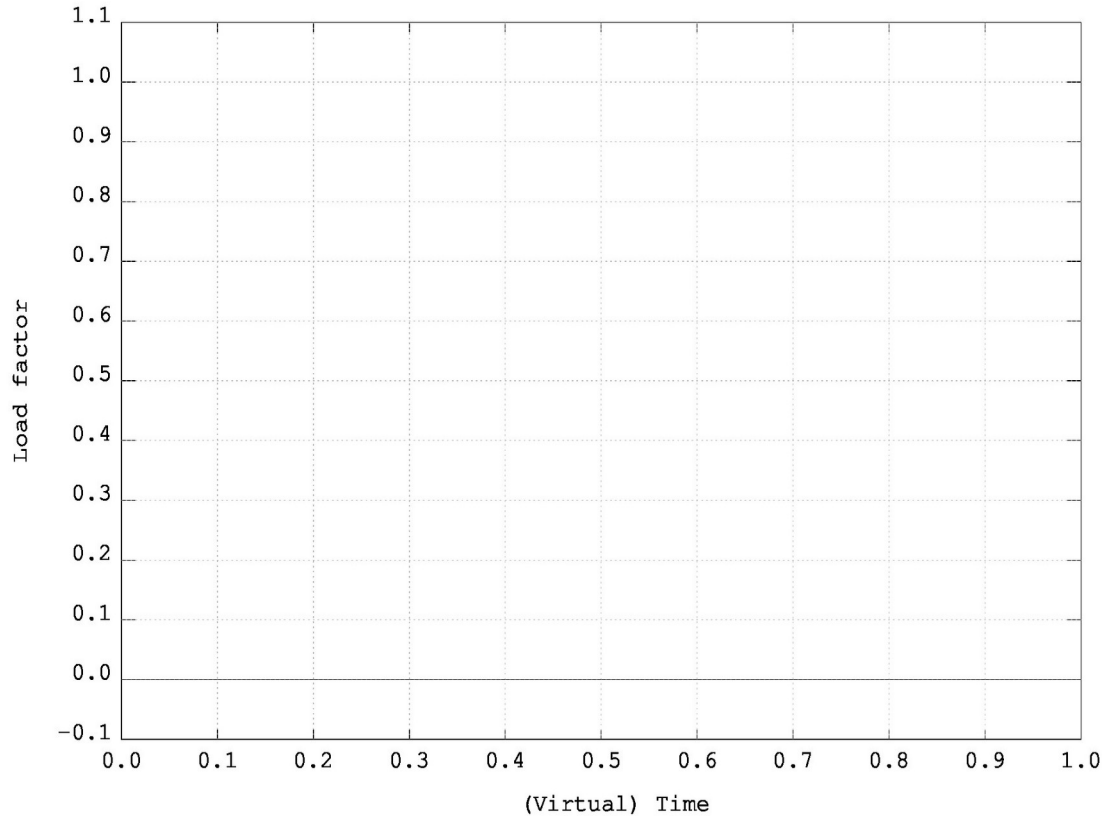
Decreasing load multiplier function with equilibrium iteration



Load decrease multiplier

Benchmark Problem for the Stability of a Retaining Wall – Decreasing load
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

Zero load multiplier function



Load zero multiplier

Benchmark Problem for the Stability of a Retaining Wall – Zero load

DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem:

Material model and parameters

Material model:

**Drucker-Prager ideal elasto-plasticity
without hardening or softening**

Constant stiffness parameters

**Adjustment of strength parameters
to the average of Mohr-Coulomb
compression and tension**

Drucker-Prager strength parameters (?):

$$\text{ALFA} = \text{ETA} = \frac{2 \sqrt{3} \sin(\varphi)}{9 - \sin^2(\varphi)}, \quad \text{BETA} = \text{MU} = \sqrt{\frac{2}{3}} (?),$$

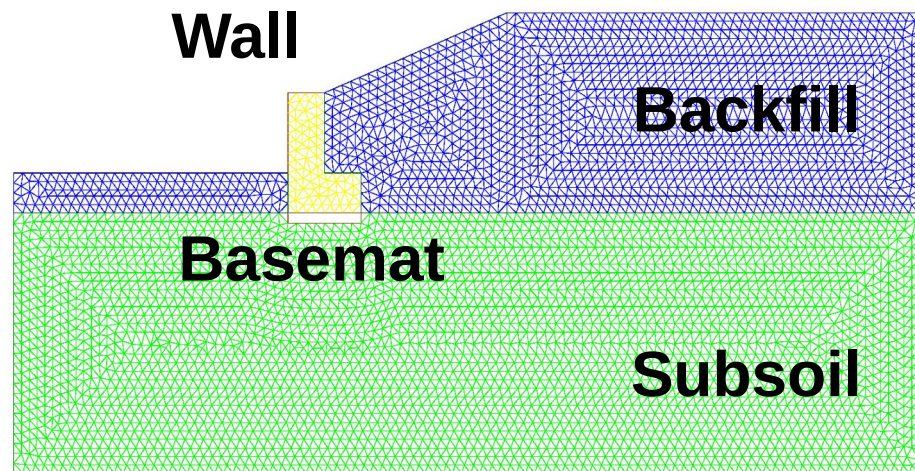
$$\text{K} = \text{KL} = \frac{6 \sqrt{3} c \cos(\varphi)}{9 - \sin^2(\varphi)},$$

$$\text{GAMM} = \frac{2 \sqrt{3} \sin(\psi)}{9 - \sin^2(\psi)}, \quad \text{DELT} = \sqrt{\frac{2}{3}} (?)$$

Solution of the Benchmark Problem: Geometry and mesh

Soil clusters and mesh

Cast3M

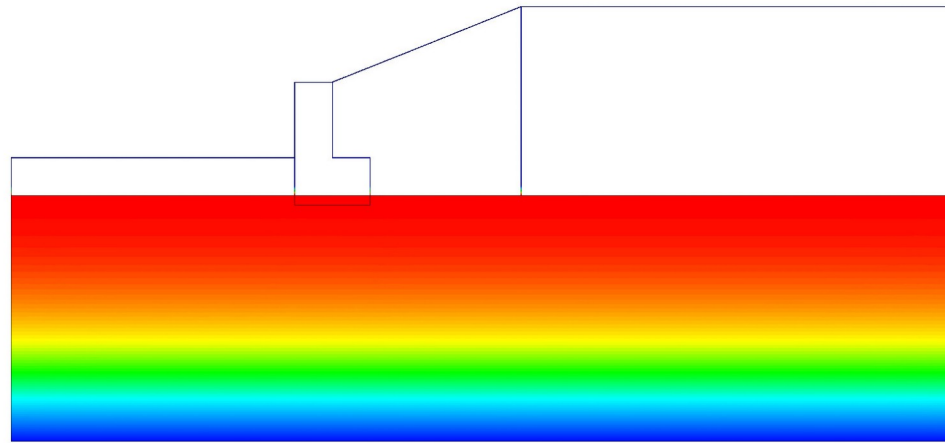


Soil clusters and mesh

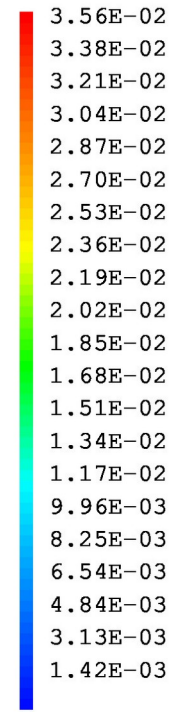
Retaining Wall – Geometry, soil clusters and wall – Subsoil, basemat, wall and backfill
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem: Initial stress state (SLS)

SLS: Subsoil



UTOT
 < 3.58E-02
 > 0.00E+00



Cast3M
SiSi

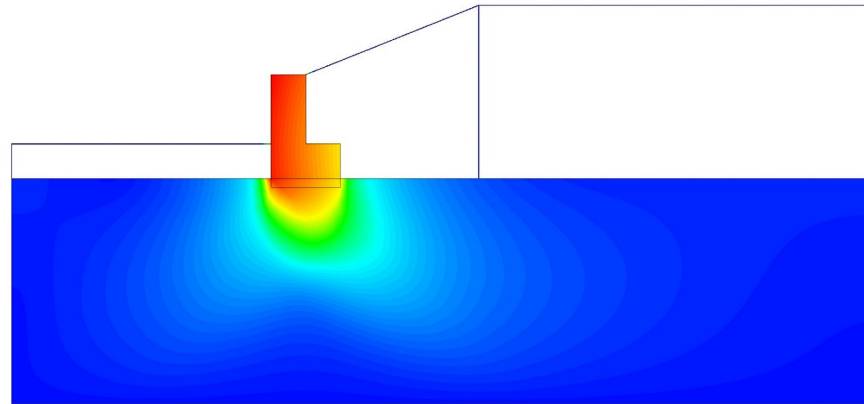
Phase 1: Total displacement amplitudes (SLS)

AMPLITUDE
 DEFORMEE

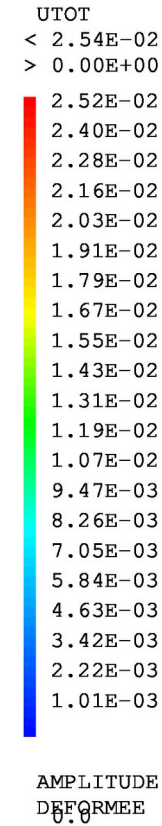
Retaining wall – Total displacement amplitudes in the subsoil under self weight
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem: Construction of the retaining wall (SLS)

SLS: Retaining wall



Phase 2: Total displacement amplitudes (SLS)

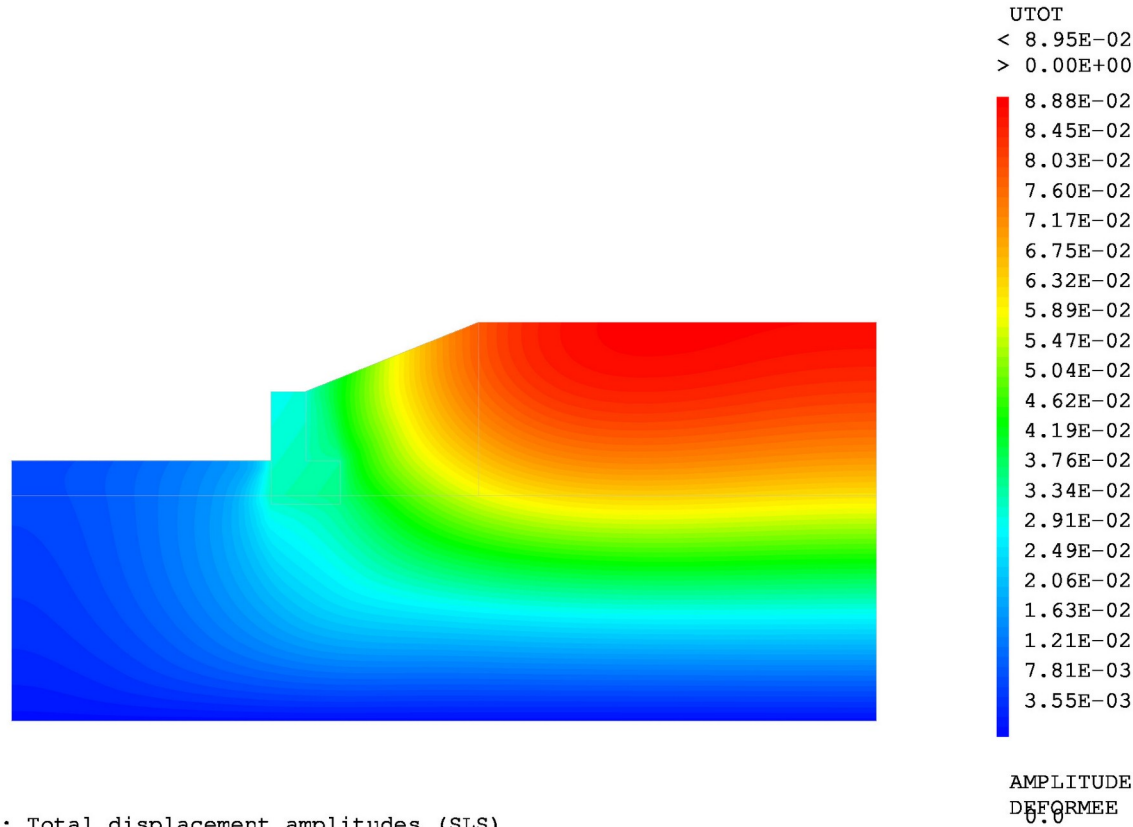


Cast3M
SiSi

Retaining wall – Total displacement amplitudes after construction of the retaining wall
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem: Backfill behind the retaining wall (SLS)

SLS: Retaining wall



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SiSi

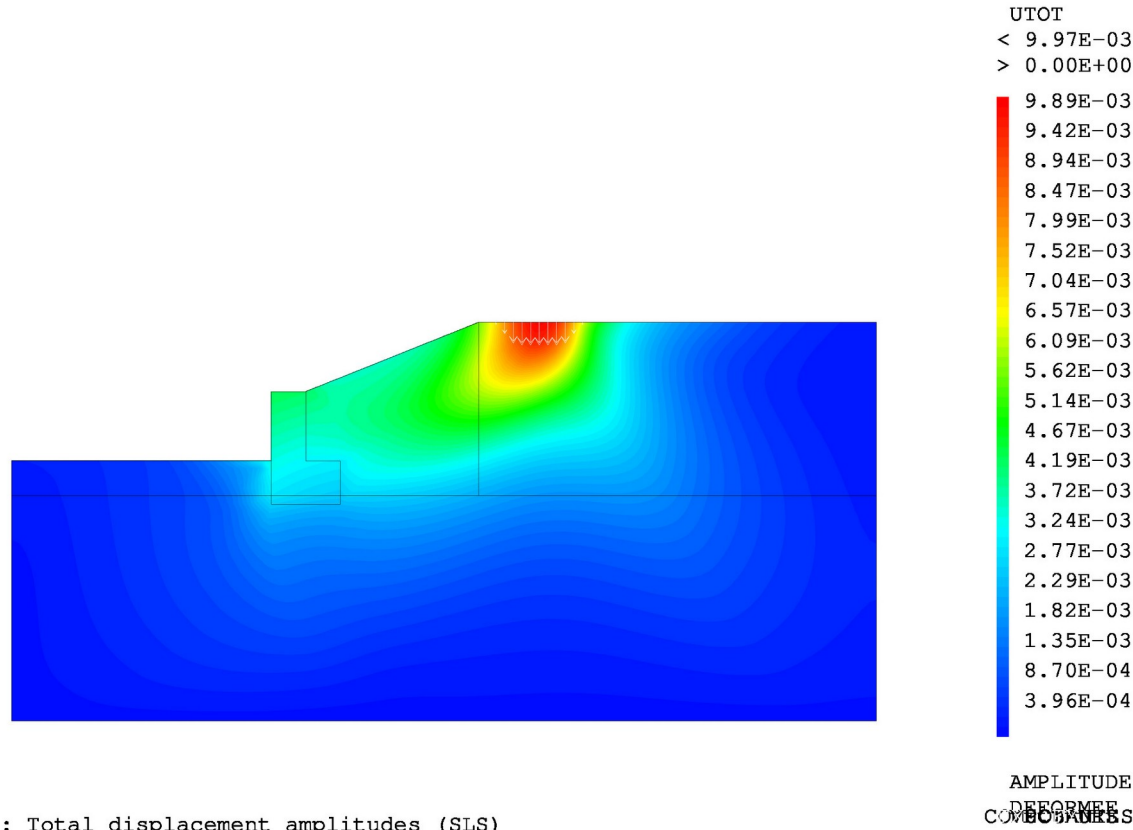
Phase 3: Total displacement amplitudes (SLS)

Retaining wall – Total displacement amplitudes after backfill behind the retaining wall
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem: Imposition of external load (SLS)

SLS: External load

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SiSi



Phase 4: Total displacement amplitudes (SLS)

Retaining wall – Total displacement amplitudes under imposed external load
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem:

The strength reduction method (ULS, GEO-3)

Strength reduction method:

Global safety factor concept: η

$$r_n = \left\| \mathbf{f} - \mathbf{K} \left(\varphi_{k,mob}, c_{k,mob}, \psi_{k,mob}, \sigma_{t,mob} \right) \cdot \mathbf{u}_n \right\| < \epsilon$$

$$\tan \varphi_{k,mob} = \frac{\tan \varphi_k}{\eta}, \quad c_{k,mob} = \frac{c_k}{\eta}, \quad \tan \psi_{k,mob} = \frac{\tan \psi_k}{\eta}, \quad \sigma_{t,mob} = \frac{\sigma_{t,k}}{\eta}$$

Partial safety factor concept: $\mu, (\gamma)$

$$r_n = \left\| \mathbf{f} - \mathbf{K} \left(\varphi_{d,mob}, c_{d,mob}, \psi_{d,mob}, \sigma_{t,d,mob} \right) \cdot \mathbf{u}_n \right\| < \epsilon$$

$$\tan \varphi_{d,mob} = \mu \tan \frac{\varphi_k}{\gamma_\varphi}, \quad c_{d,mob} = \mu \frac{c_k}{\gamma_c}, \quad \tan \psi_{d,mob} = \mu \frac{\tan \psi_k}{\gamma_\psi}, \quad \sigma_{t,d,mob} = \mu \frac{\sigma_{t,k}}{\gamma_{\sigma,t}}$$

Special case of the partial safety factors and dimensioning values of the strength parameters for analyses on the strength reduction method behaviour:

$$\gamma_{\varphi} = \gamma_c = \gamma_{\psi} = \gamma_{\sigma,t} = 1.0$$

$$\tan \varphi_d = \tan \varphi_k \quad , \quad c_d = c_k \quad , \quad \psi_d = \psi_k \quad , \quad \sigma_{t,d} = \sigma_{t,k}$$

Identity for the global safety factor and the utilisation factor under these conditions:

$$\mu = \frac{1}{\eta} \quad , \quad \eta = \frac{1}{\mu}$$

Strength reduction method:

Analysis is based on the limit equilibrium of external and internal forces in the model

Trivial solution: gradual reduction of the mobilized strength (manually or in a loop)

Advanced solution: recursive reduction of the mobilized strength using a search strategy based on a convergence criterion

Strength reduction method:

Restrictions on suitable material behaviour

Drucker-Prager, Mohr-Coulomb and Hoek-Brown ideal elasto-plasticity are considered to be adequate

Material models describing more sophisticated physics must be replaced and transformed before analysis phase

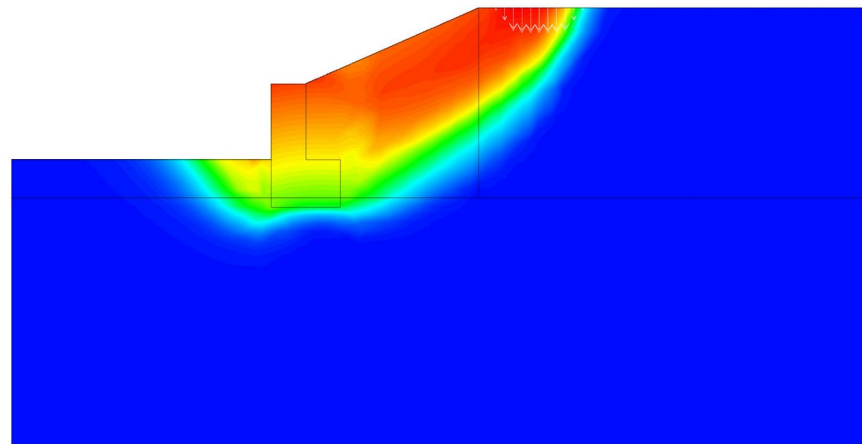
Solution of the Benchmark Problem:

General stability analysis with the strength reduction method (ULS, GEO-3)

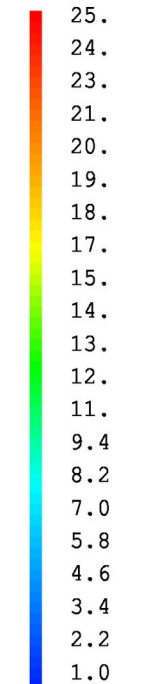
ULS, Geo-3: Retaining wall displacements

$$\eta = 1.41 [1]$$

$$\mu = 0.71 [1]$$



UTOT
 < 2.52E+01
 > 0.00E+00



Cast3M
SiSi

Phase 5: Total displacement amplitudes (ULS)

AMPLITUDE
 DEFORMEE
 COMBIBARS

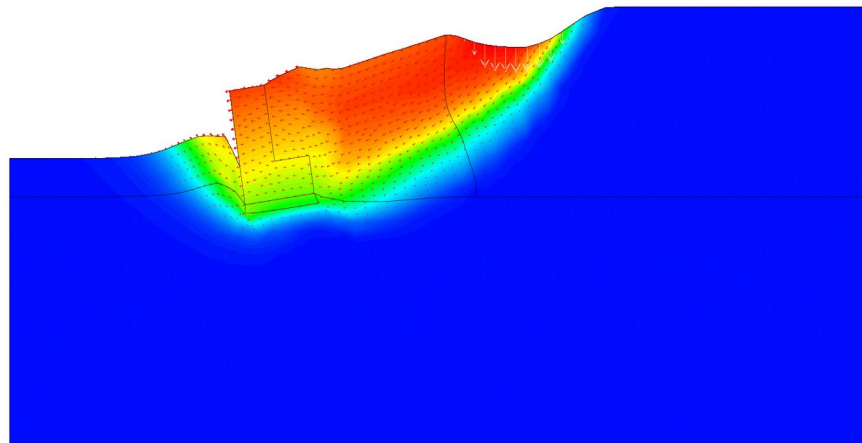
Retaining wall – Total displacements in ultimate limit state of the case SiSi

DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

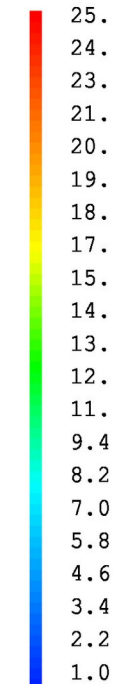
ULS, Geo-3: Retaining wall failure kinematics

$$\eta = 1.41 [1]$$

$$\mu = 0.71 [1]$$



UTOT
 < 2.52E+01
 > 0.00E+00



Cast3M
SiSi

Phase 5: Total displacement amplitudes (ULS)

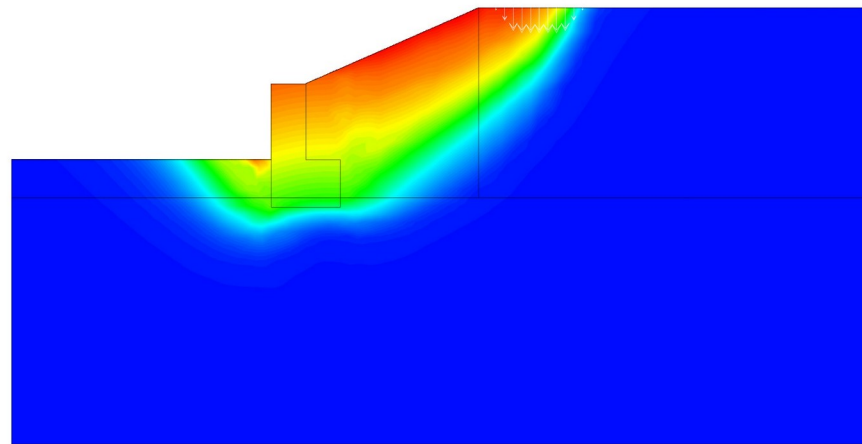
AMPLITUDE
 DEFORMEE
 CONTOURS

Retaining wall – Kinematics and total displacements in ultimate limit state of the case SiSi
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

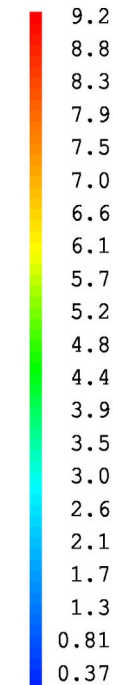
ULS, Geo-3: Retaining wall displacements

$$\eta = 1.56 [1]$$

$$\mu = 0.64 [1]$$



UTOT
 < 9.30E+00
 > 0.00E+00



Cast3M
SiSa

Phase 5: Total displacement amplitudes (ULS)

AMPLITUDE
 DEFORMEE
 COMBINAISON

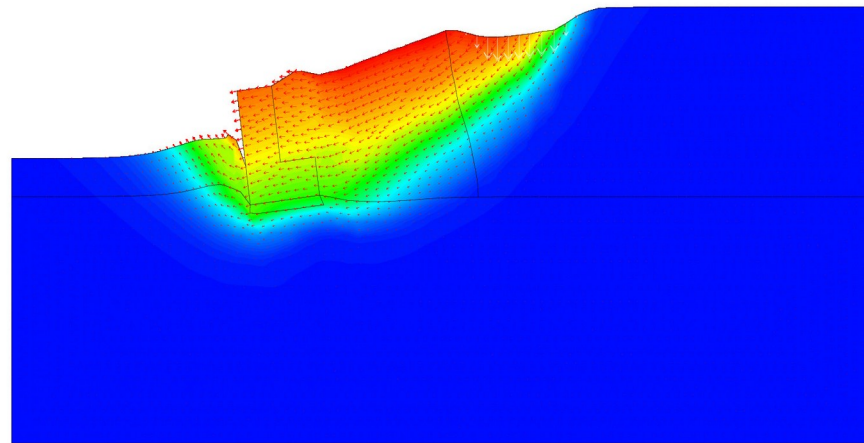
Retaining wall – Total displacements in ultimate limit state of the case SiSa

DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

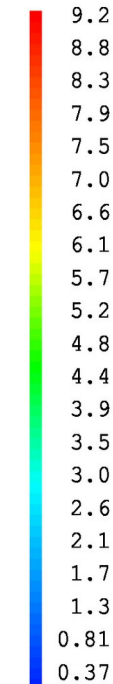
ULS, Geo-3: Retaining wall failure kinematics

$$\eta = 1.56 [1]$$

$$\mu = 0.64 [1]$$



UTOT
 < 9.30E+00
 > 0.00E+00



Cast3M
SiSa

Phase 5: Total displacement amplitudes (ULS)

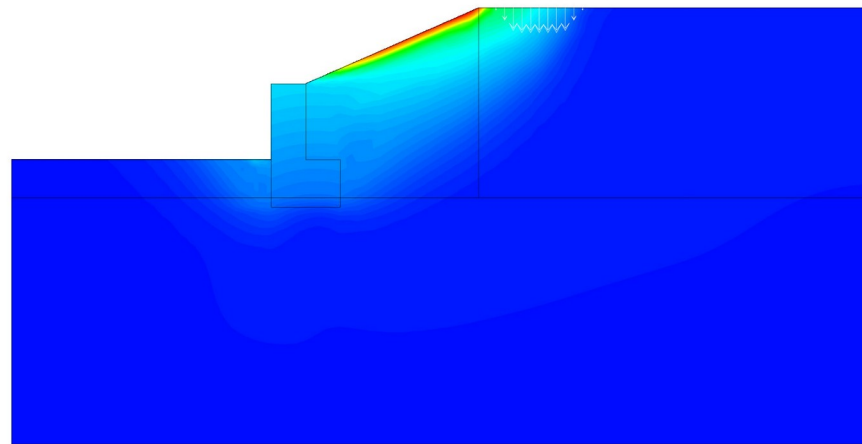
AMPLITUDE
 DEFORMEE
 CONTOURS

Retaining wall – Kinematics and total displacements in ultimate limit state of the case SiSa
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

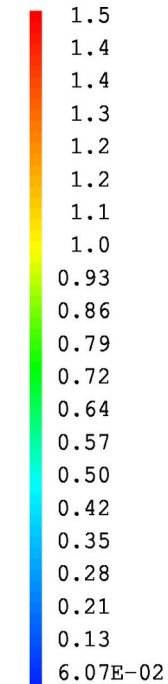
ULS, Geo-3: Retaining wall displacements

$$\eta = 1.74 [1]$$

$$\mu = 0.57 [1]$$



UTOT
 < 1.53E+00
 > 0.00E+00



Cast3M
SaSa

Phase 5: Total displacement amplitudes (ULS)

AMPLITUDE
 DEFORMEE
 COMBIBARS

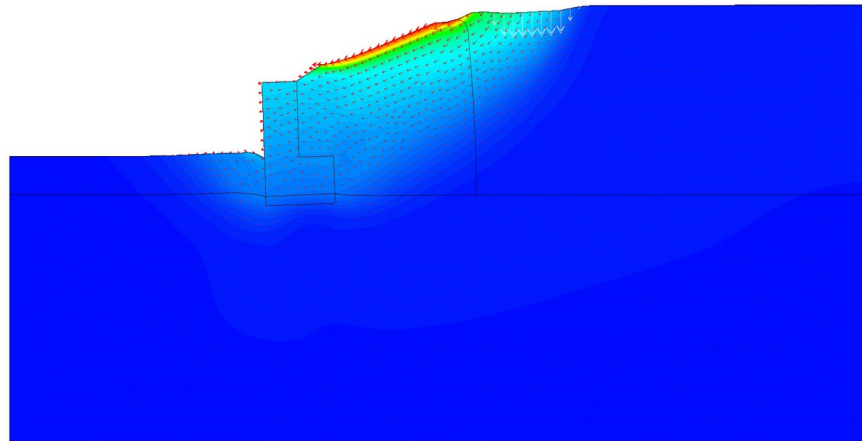
Retaining wall – Total displacements in ultimate limit state of the case SaSa

DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

ULS, Geo-3: Retaining wall failure kinematics

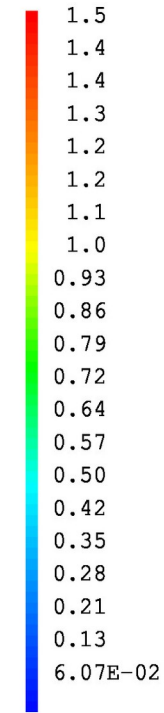
$$\eta = 1.74 [1]$$

$$\mu = 0.57 [1]$$



Phase 5: Total displacement amplitudes (ULS)

UTOT
 < 1.53E+00
 > 0.00E+00



AMPLITUDE
 DEFORMEE
 COMPARSS

Cast3M
SaSa

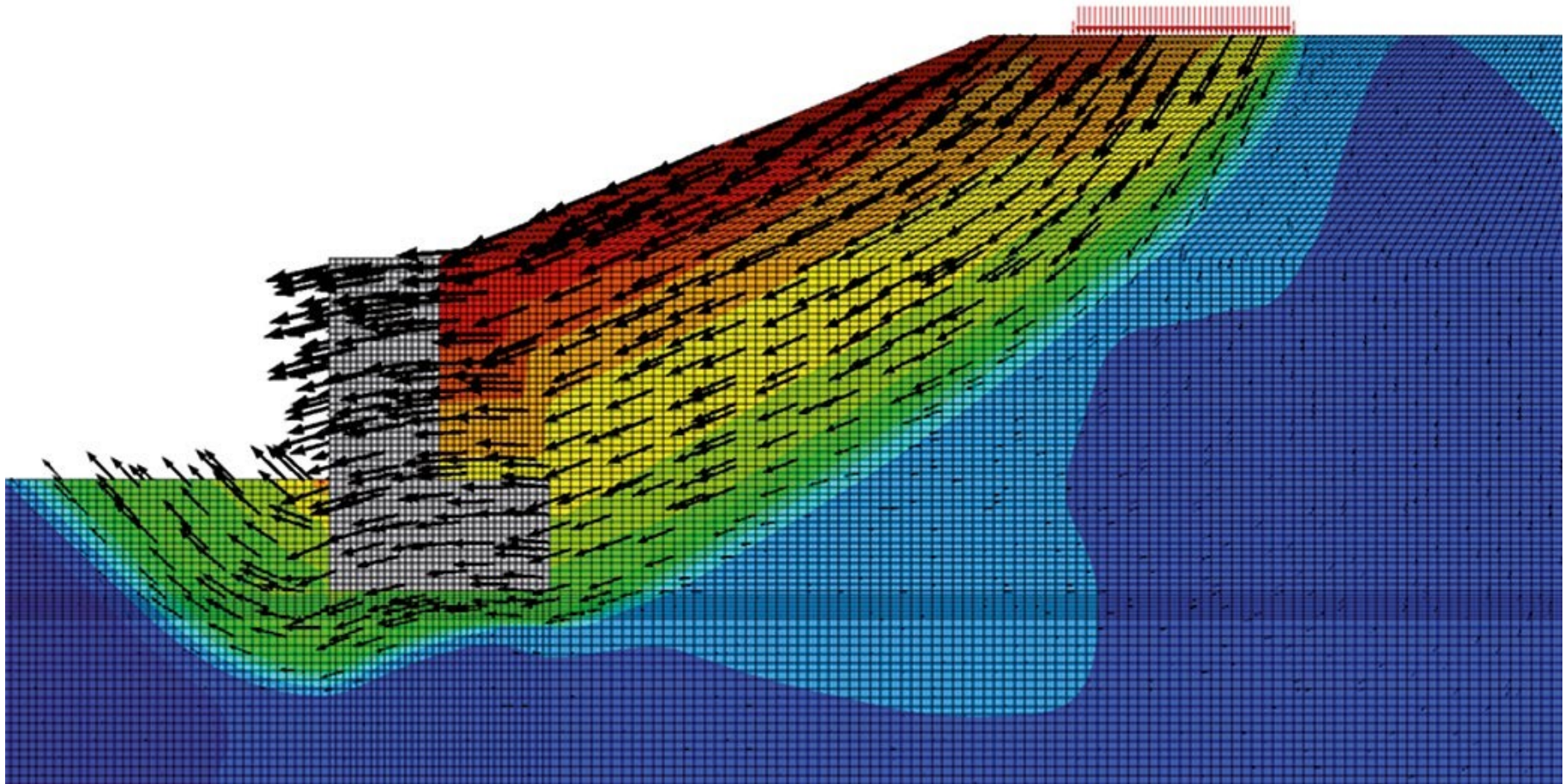
Retaining wall – Kinematics and total displacements in ultimate limit state of the case SaSa
 DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem:

**General stability analysis with
the strength reduction method
(ULS, GEO-3)**

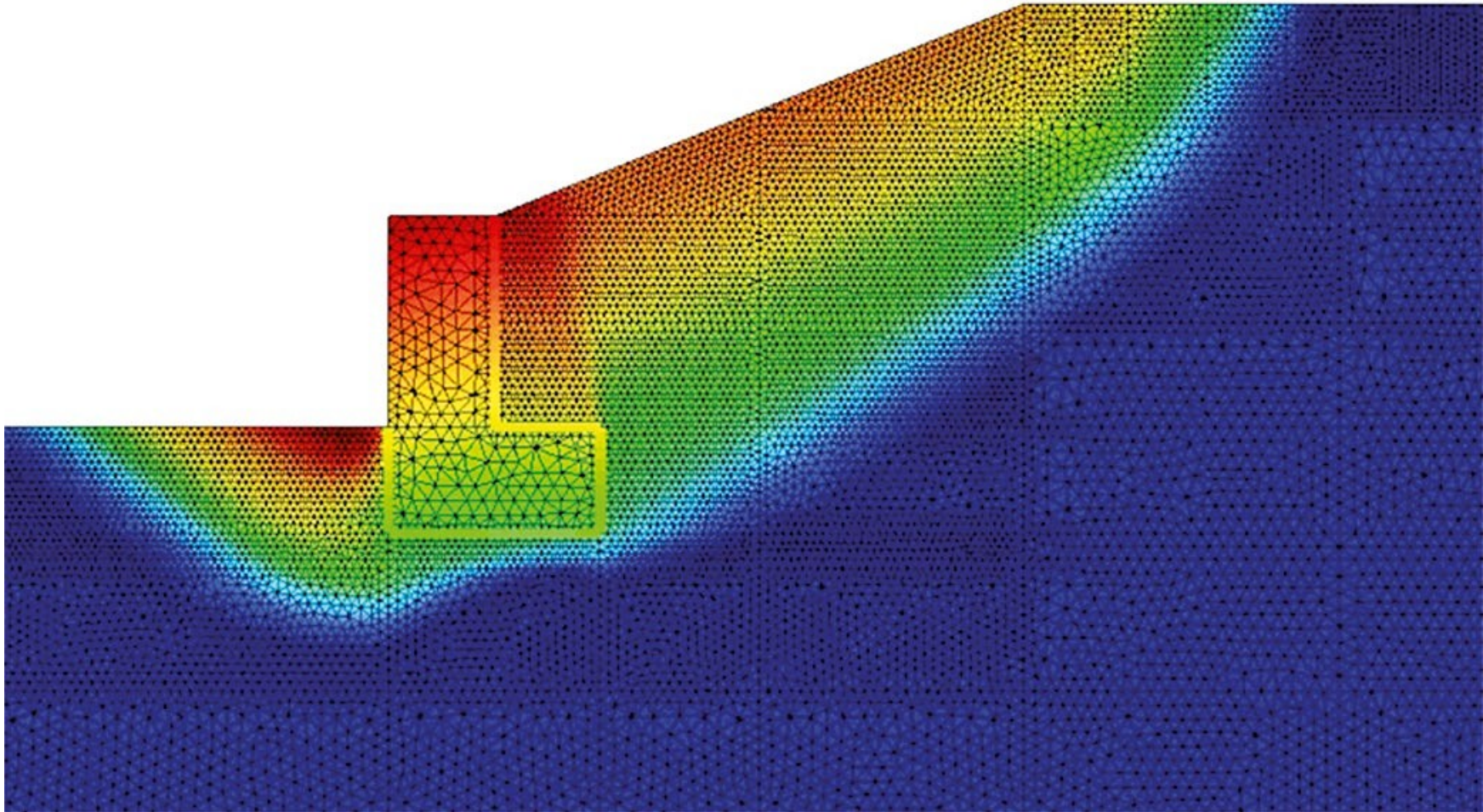
Alternative solution examples

ULS, Geo-3: Failure kinematics, Flac3D



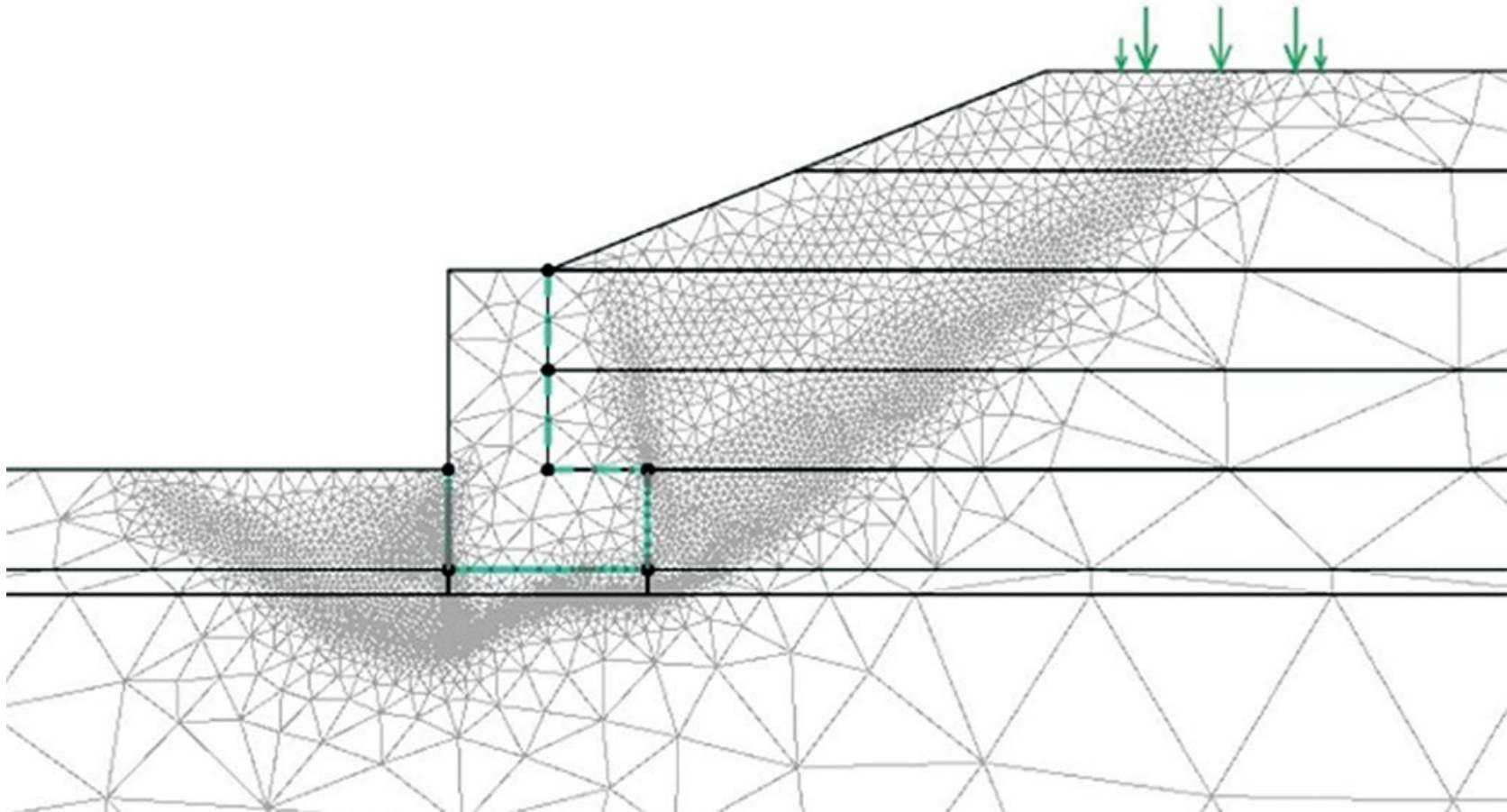
Retaining wall – Total displacements in ultimate limit state with the code Flac3D (FDM)
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

ULS, Geo-3: Failure kinematics, Tochnog



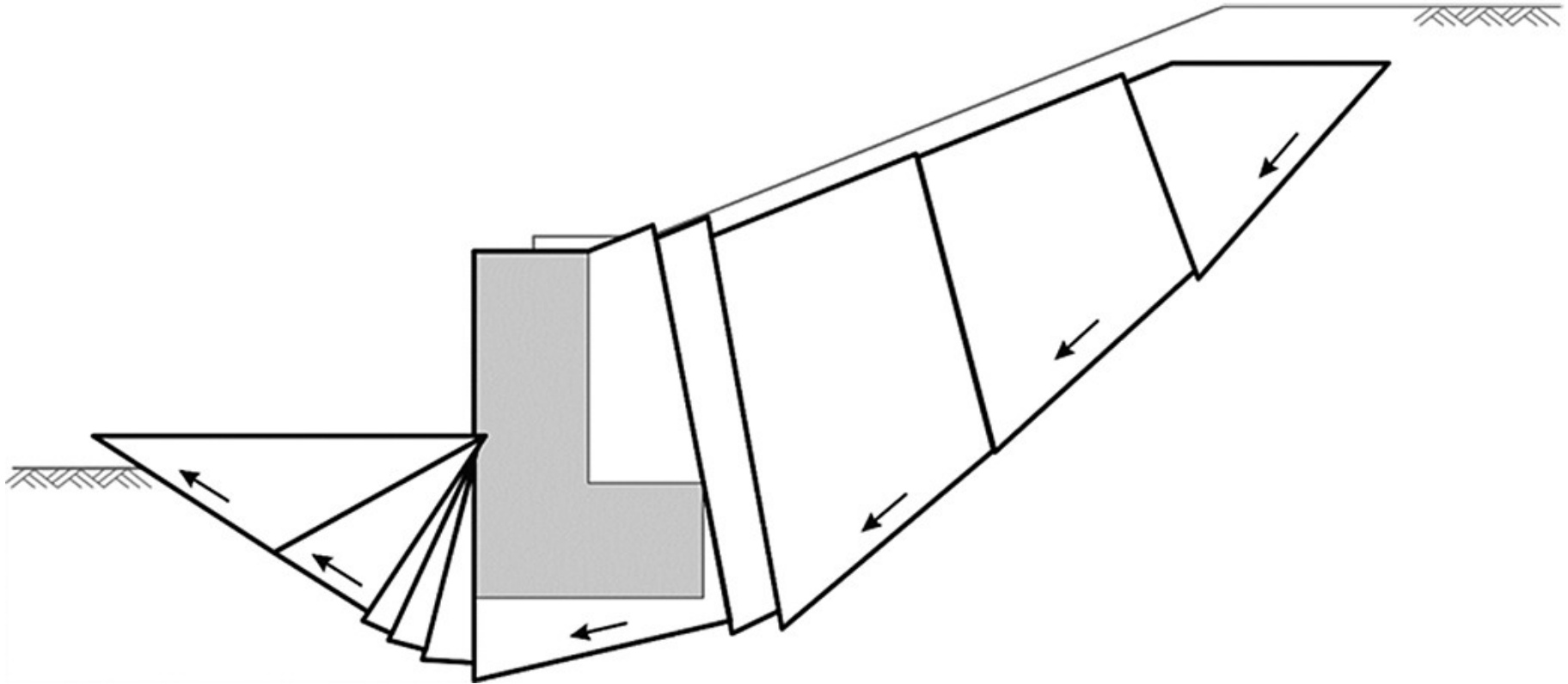
Retaining wall – Total displacements in ultimate limit state with the code Tochnog (FEM)
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

LEM: Limit Equilibrium Method



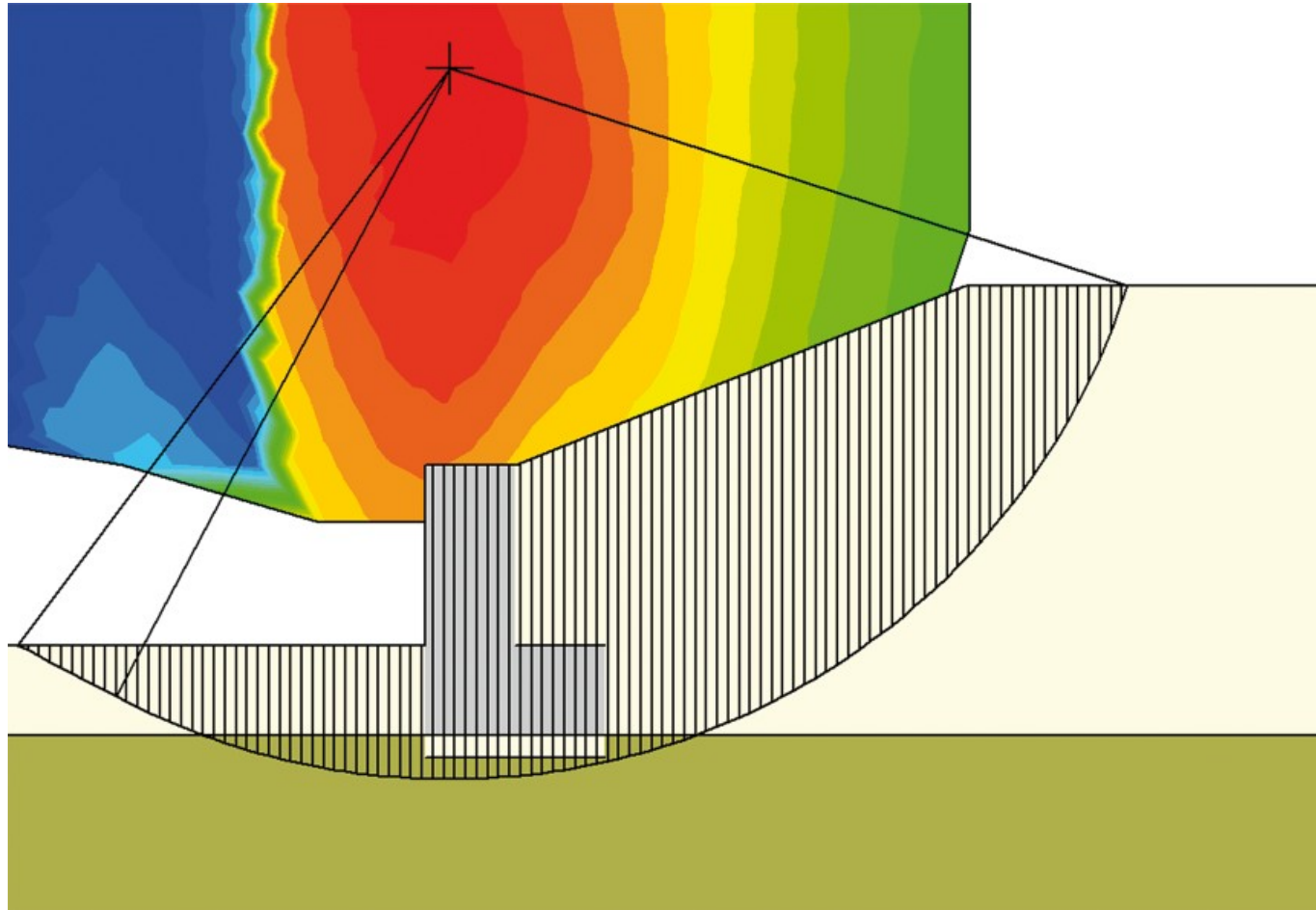
Retaining wall – Ultimate limit state analysis with the Limit Equilibrium Method (LEM)
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

KEM: Kinematic Element Method



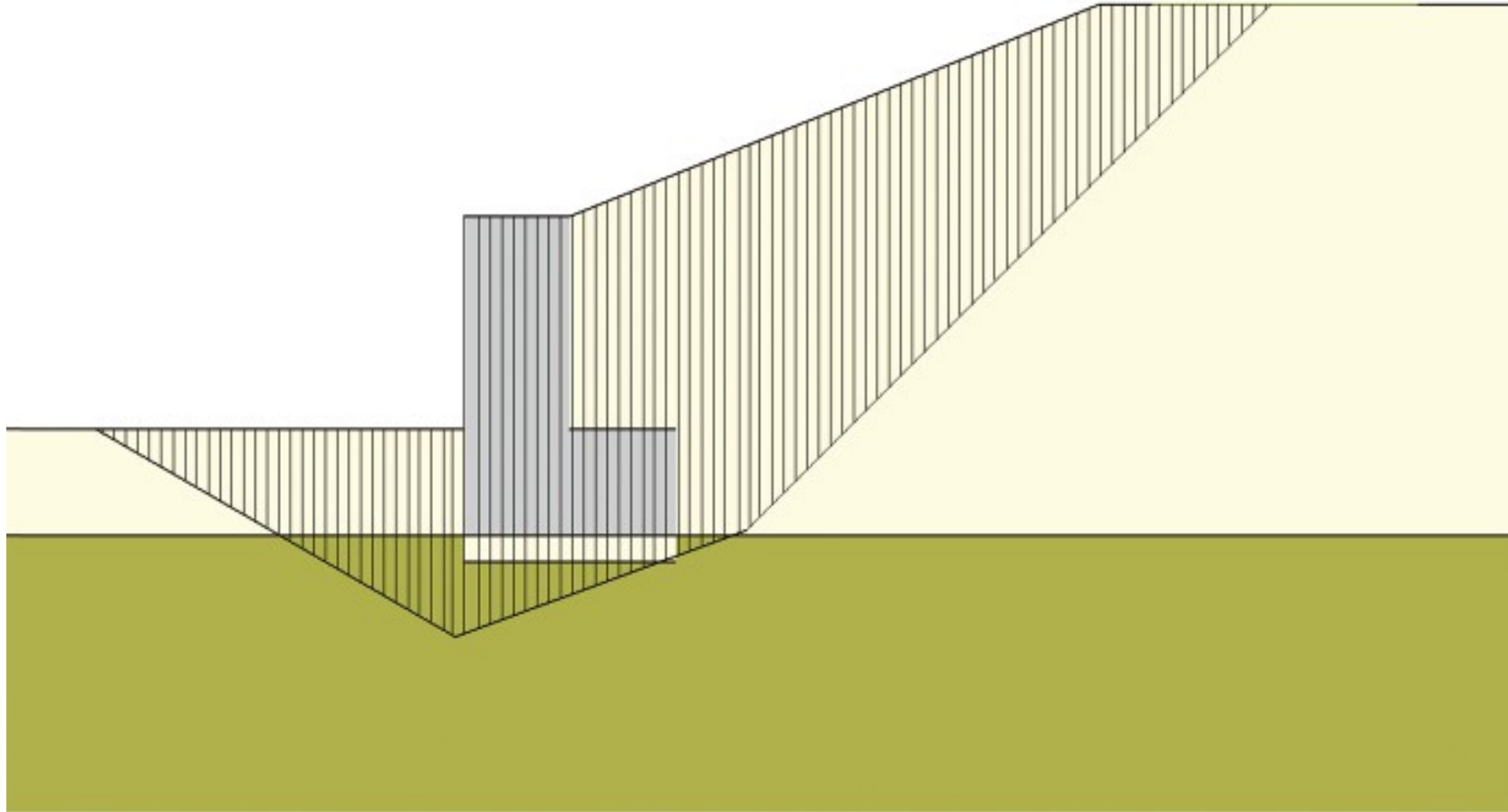
Retaining wall – Ultimate limit state analysis with the Kinematic Element Method (KEM)
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; *Geotechnik* 42(2019), no.2, p.88-97.

Method of slices with circular slip surface



Retaining wall – Ultimate limit state analysis with the method of slices (e.g. Bishop method)
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Method of slices with polygonal slip surface



Retaining wall – Ultimate limit state analysis with the method of slices (e.g, Janbu method)
DGGT e.V., AK1.6: Stability Calculation with FEM with Strength Parameter Reduction; Geotechnik 42(2019), no.2, p.88-97.

Solution of the Benchmark Problem:

**General stability analysis with
the strength reduction method
(ULS, GEO-3)**

Results

Global safety factors /utilisation factors/

| Solver:1 | Method: | SiSi: | SiSa: | SaSa: |
|----------------|------------|---------------------------------------|---------------------------------------|---------------------------------------|
| Plaxis | FEM | 1.36 | 1.56 | 1.78 |
| Z-Soil | FEM | 1.36 | 1.58 | 1.78 |
| Tochnog | FEM | 1.41 | 1.56 | 1.74 |
| Sofistik | FEM | 1.40 | 1.61 | 1.83 |
| Flac3D | FDM | 1.41 | 1.63 | 1.83 |
| Optum G2 | LEM | 1.34-1.37 | 1.50-1.60 | 1.79-1.83 |
| KEM | KEM | 1.43 | 1.58 | 1.93 |
| Bishop / Janbu | Slices | 1.58 / 1.24 | 1.89 / 1.44 | 2.19 / 1.71 |
| Cast3M | FEM | (1.41) /μ=0.71/ | (1.56) /μ=0.64/ | (1.74) /μ=0.57/ |



Summary and conclusions

Geotechnical problems:

Construction in (many) individual phases

Increasing load factors for material placement (fill) and decreasing load factors for material removal (excavation) taking stiffness modelling correctly into account

Constant load factors for existing and zero load factors for future charges

Geotechnical problems:

**Serviceability limit state (SLS)
computations for deformation analysis**

**Ultimate limit state (ULS)
computations for stability analysis**

**Strength reduction method is a
powerful numerical technique
implementing different safety concepts**

Cast3M in Geotechnical problems:

Serviceability limit state (SLS) analyses deliver adequate quantitative results

Ultimate limit state (ULS) analyses deliver adequate failure kinematics

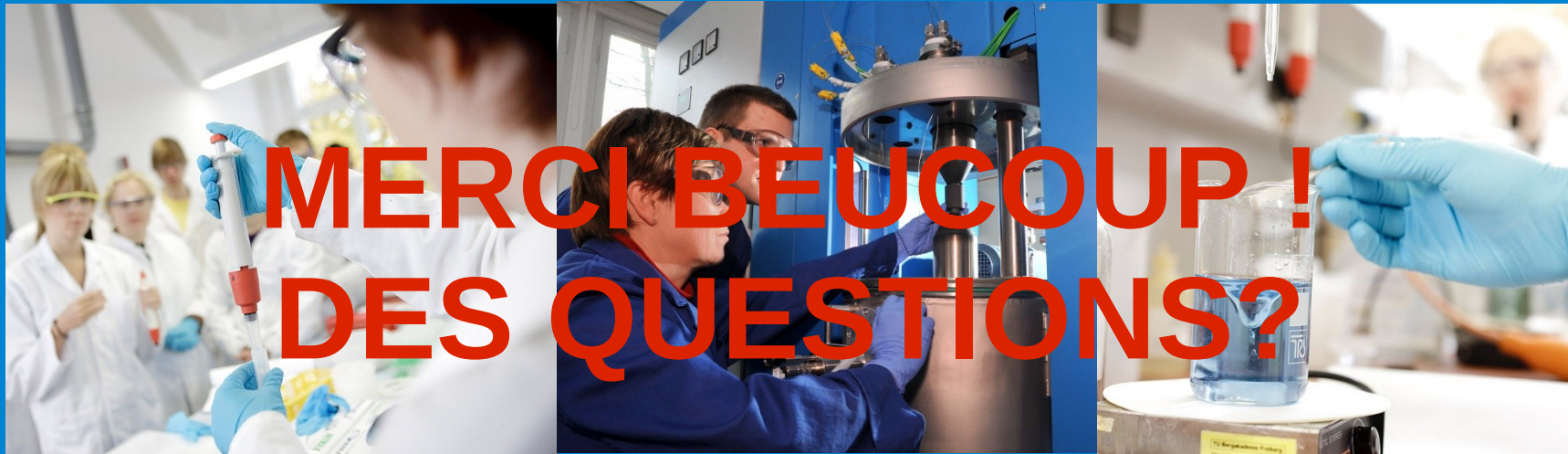
Ultimate limit state (ULS) analyses should be considered as a **promiseful experimental feature for the moment**



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