

Politecnico di Milano  
Dipartimento di Meccanica



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## **DYNAMIC ANALYSIS OF A B $\flat$ CLARINET**



Laboratoire de Mécanique des Solides  
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# B $\flat$ CLARINET

“beating” simple reed



mouthpiece

barrel

upper part

lower part

bell



**target:** dynamic analysis during sound production

# DEVELOPMENT PLAN

## computations

FEM modeling → Castem 2000

real  
eigen  
modes

{  
sole reed  
reed + mouthpiece + barrel  
reed + clarinet



*time domain  
analysis  
dynamic*

- **one-side contact reed-mouthpiece**
- **acousto-mechanical instability  
(sound production)**

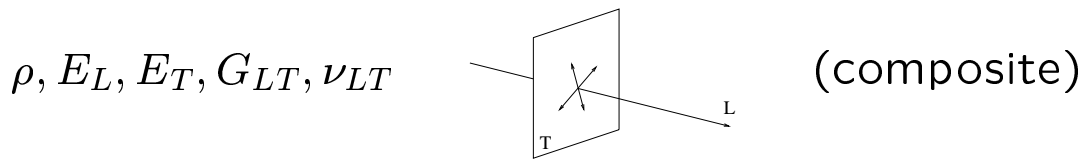


## experiments

{  
eigenfrequencies measurement  
→ *piezo-electric probe*  
eigenmodes visualization  
→ *interferometric holography*

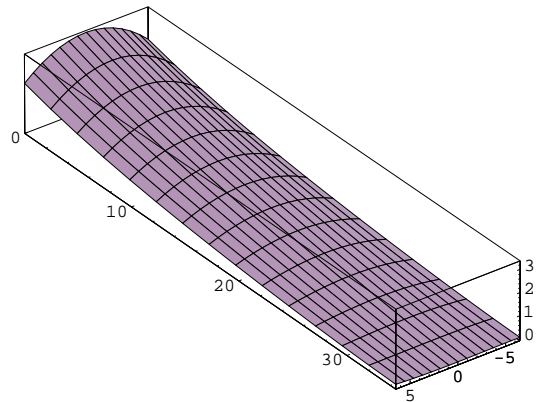
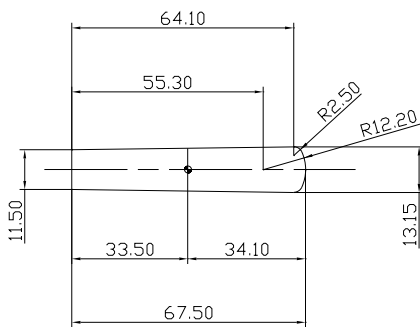
# REED

- **material:** homogeneous, elastic, transversely isotropic

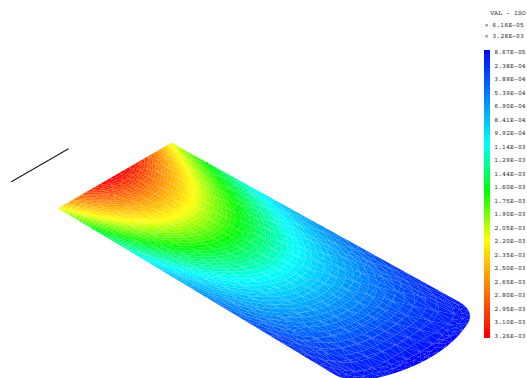


[Laine & Pinard (EP) - 1998]

- **geometry:**  $\left\{ \begin{array}{l} \text{contour} \rightarrow \textit{profil projector} \\ \text{thickness} \rightarrow \textit{CMM} + \textit{Mathematica} \end{array} \right.$



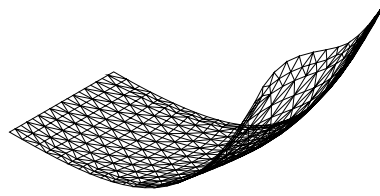
- **model:** thin plate (*Kirchhoff-Love*)



- **bond:** rigidly fixed end section (ligature)

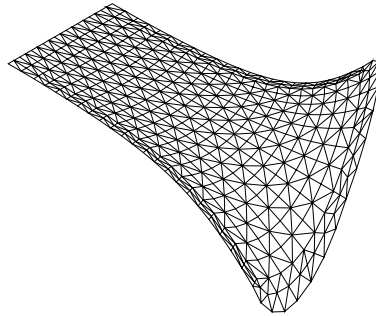
# FREE REED real eigenmodes

2312 Hz



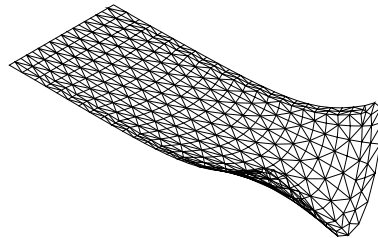
FL

3257 Hz



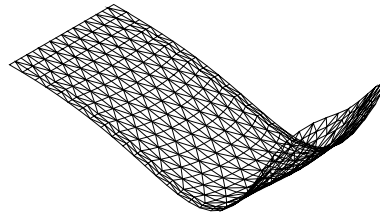
T

5840 Hz



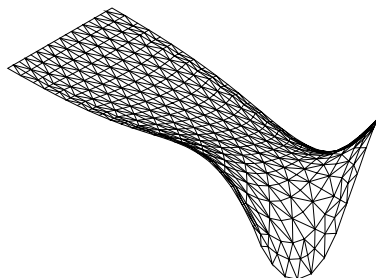
FT

6214 Hz



FL

7389 Hz

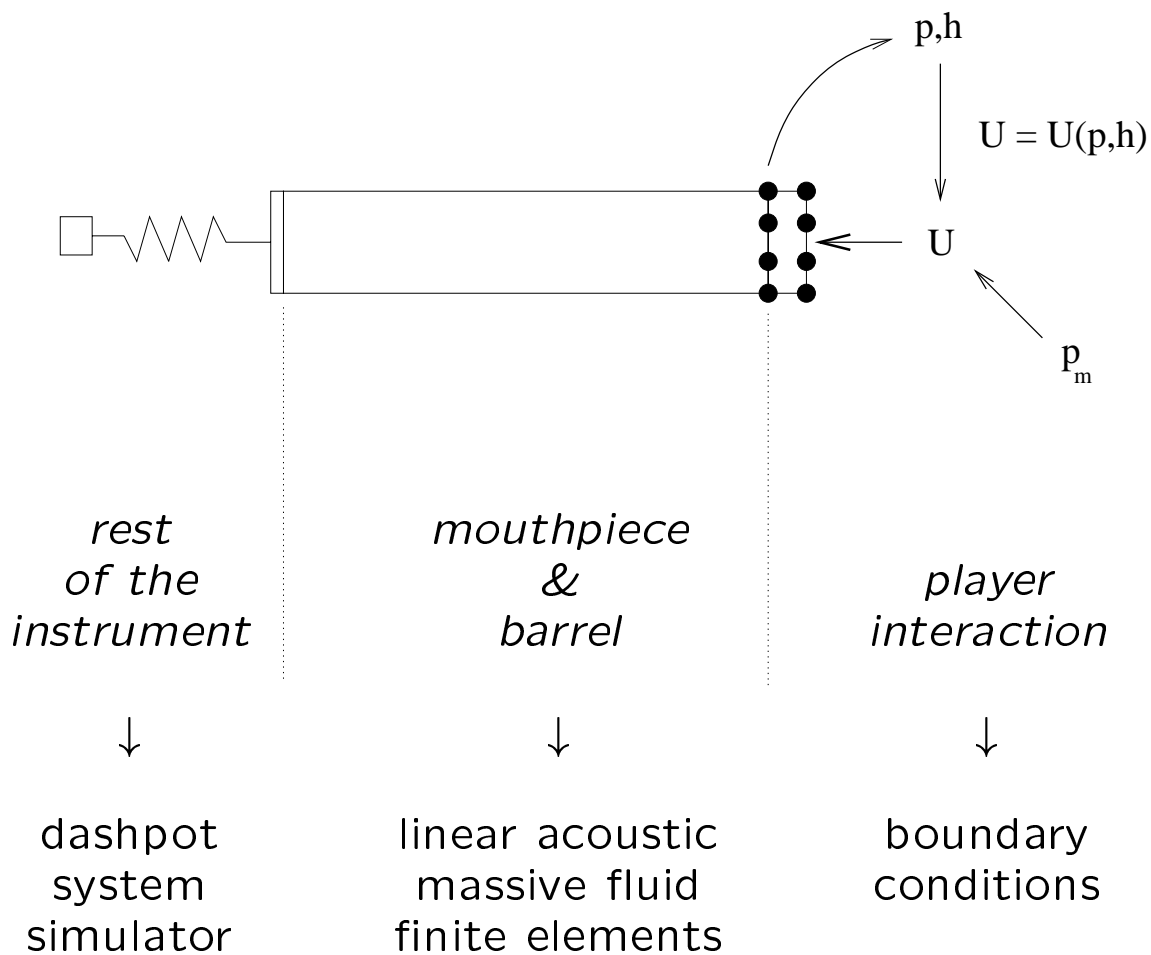


T

\*

# INSTRUMENT

**hollow object** { perfectly rigid solid boundary  
chamber = acoustic cavity



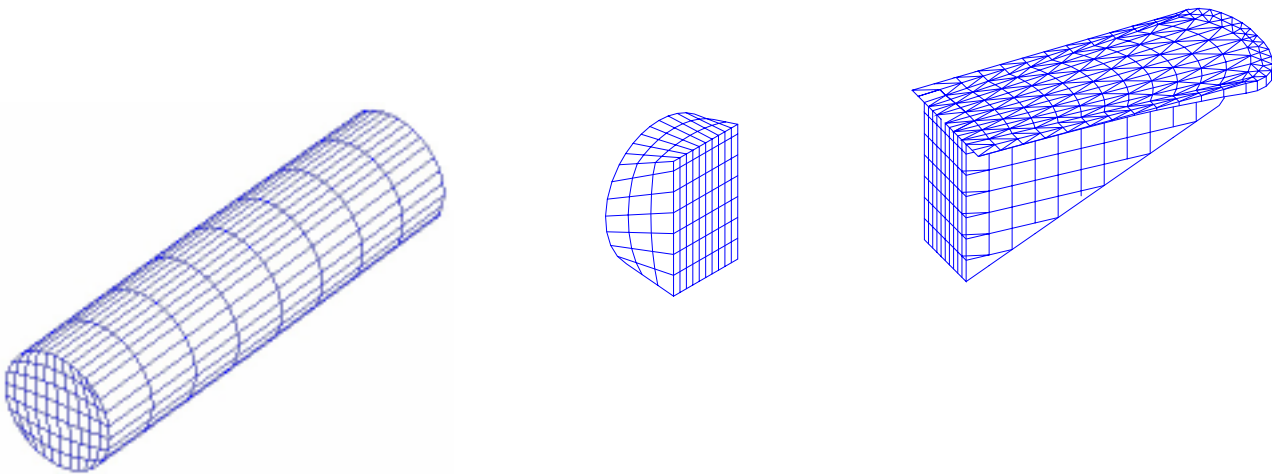
# MOUTHPIECE & BARREL

- **material:** wet air, ideal mixture of ideal gases

$$\rho, c = f(T, \varphi) \quad p_{atm}$$

$(p, U)$  perturbations with respect to equilibrium  
acoustic linear behaviour

- **geometry:** CMM + manual mesh



- **boundary conditions:**

– rigid surface:  $\frac{\partial p}{\partial n} = 0$

– free surface:  $p = 0$

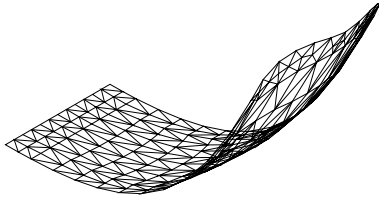
– fluid-structure interface:  $\begin{cases} \underline{U}_s \cdot \underline{n} = \underline{U} \cdot \underline{n} \\ \underline{\underline{\sigma}}_s \cdot \underline{n} = -p \underline{n} \end{cases}$

– acoustic impedance:  $Z_a(\omega) = \frac{p}{U}$

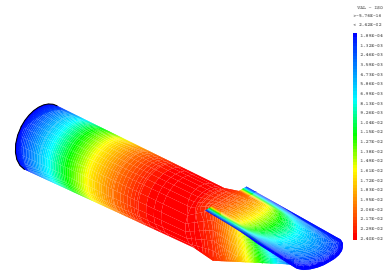
# REED + MOUTHPIECE

## real eigenmodes (1)

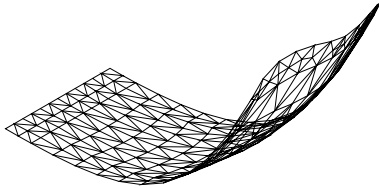
2059 Hz



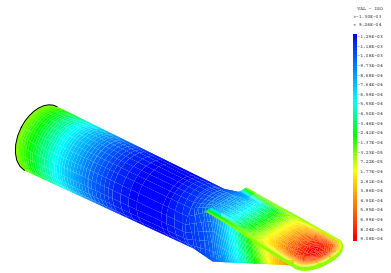
FL



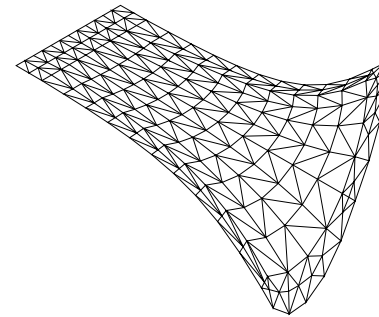
2251 Hz



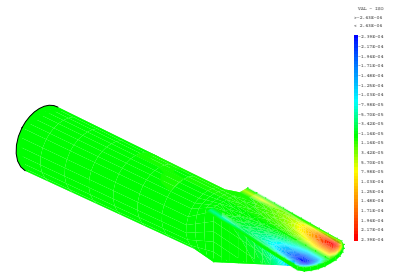
FL



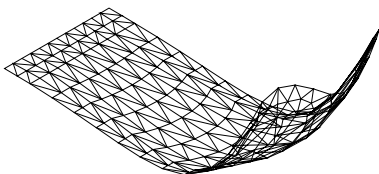
3239 Hz



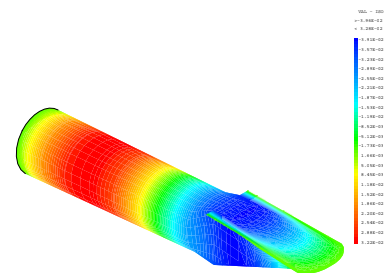
T



4496 Hz



FC

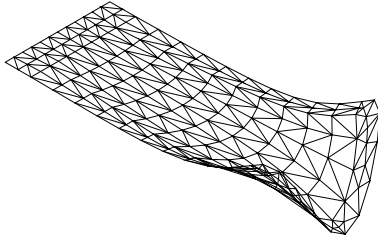




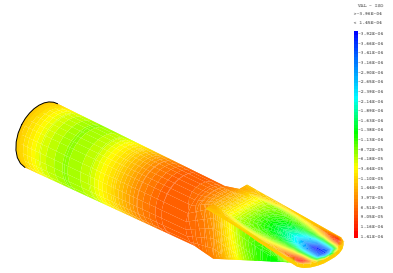
# REED + MOUTHPIECE

## real eigenmodes (2)

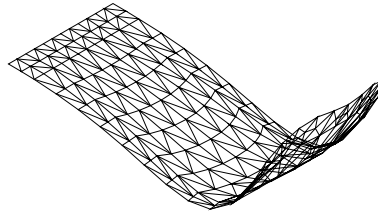
5908 Hz



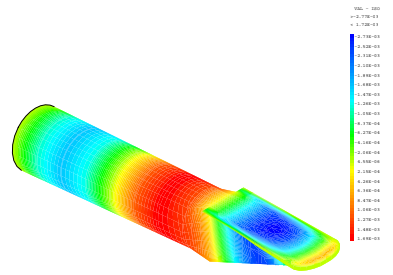
FT



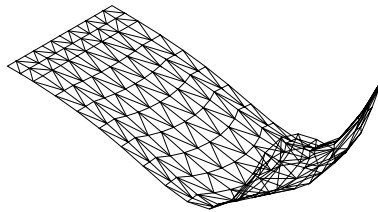
6019 Hz



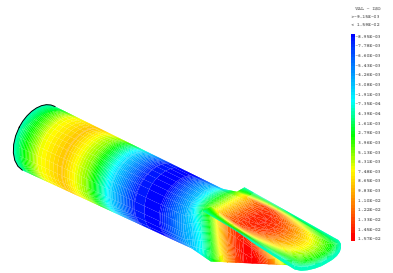
FL



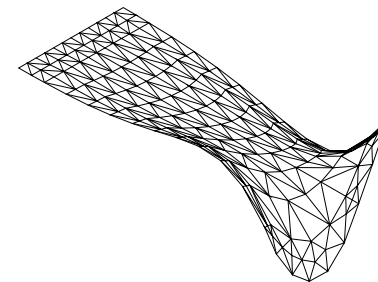
6709 Hz



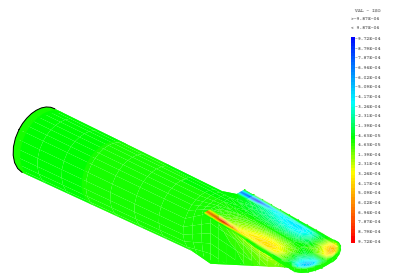
FC



7335 Hz

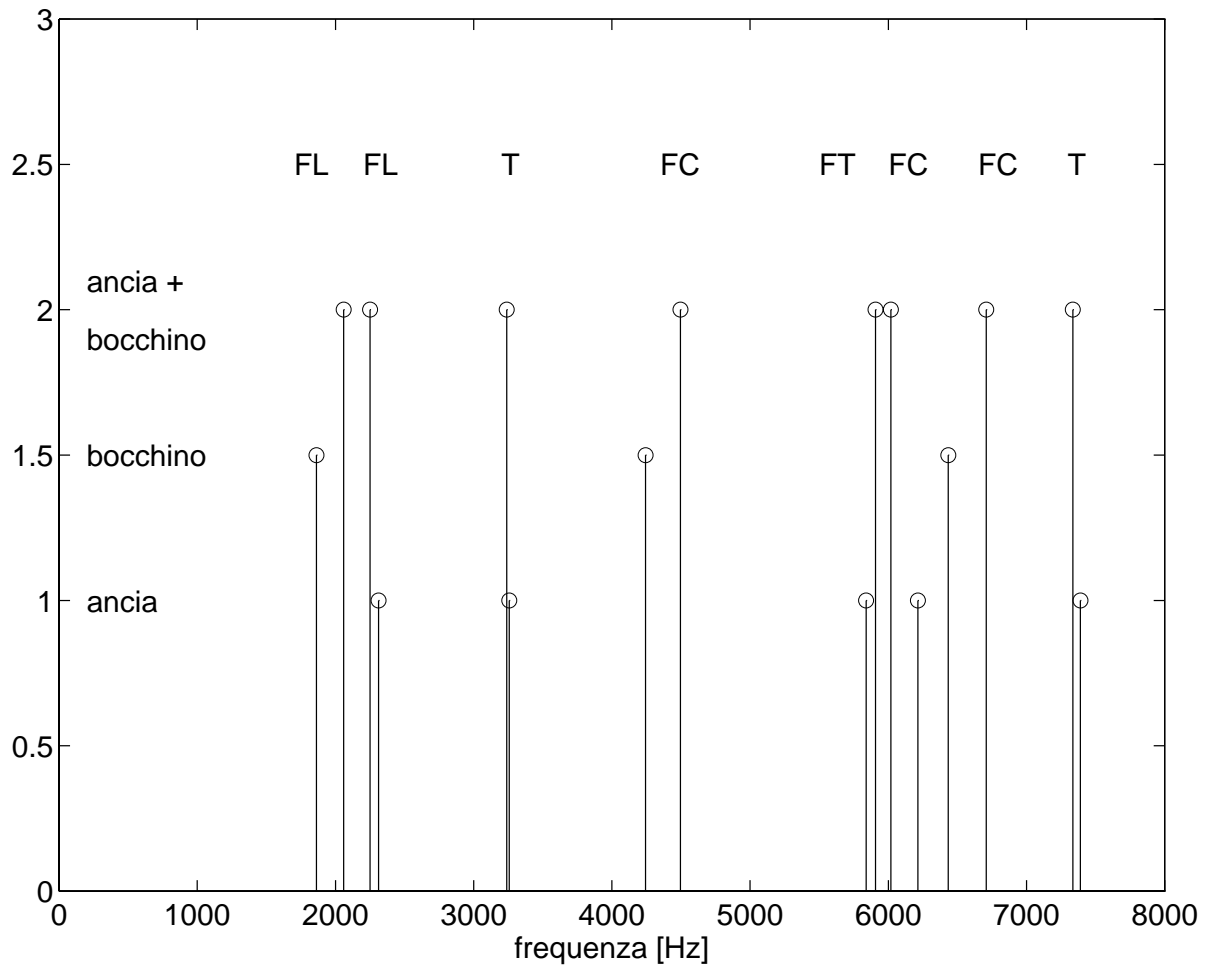


T



# COMPARISON

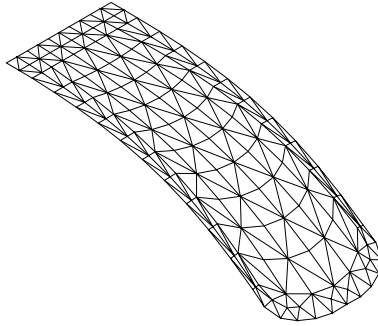
REED		REED + MOUTHPIECE		MOUTHPIECE
$f$ [Hz]	<i>eigenmode</i>	$f$ [Hz]	<i>eigenmode</i>	$f$ [Hz]
		2059	longitudinal flex	1863
2312	longitudinal flex	2251	longitudinal flex	
3257	torsion	3239	torsion	
		4496	"composite" flex	4243
5840	transversal flex	5908	transversal flex	
6214	longitudinal flex	6019	"composite" flex	
		6709	"composite" flex	6433
7389	torsion	7335	torsion	



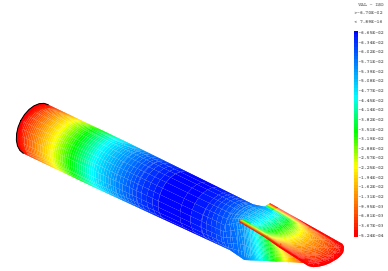
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# REED + MOUTHPIECE + BARREL real eigenmodes (1)

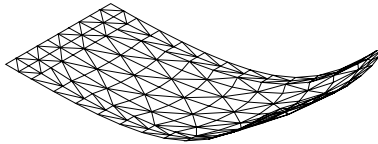
1401 Hz



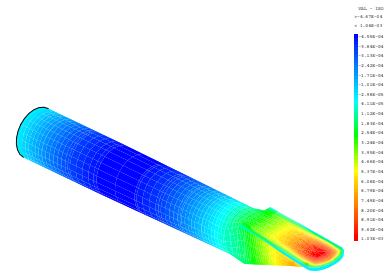
FC



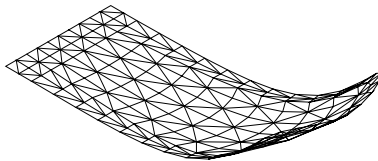
2228 Hz



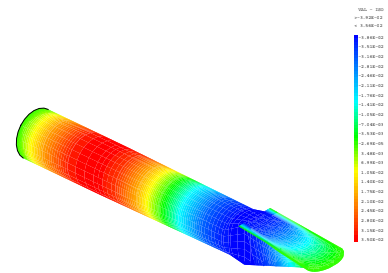
FC



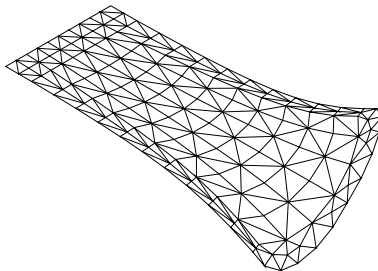
3072 Hz



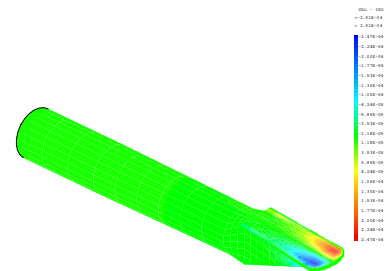
FC



3213 Hz



FC

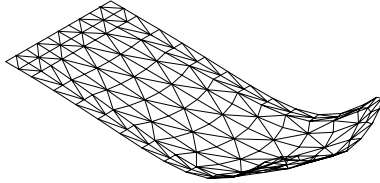


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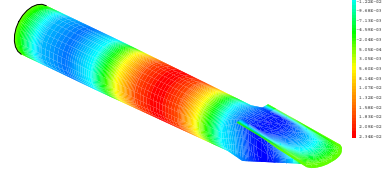
# REED + MOUTHPIECE + BARREL

## real eigenmodes (2)

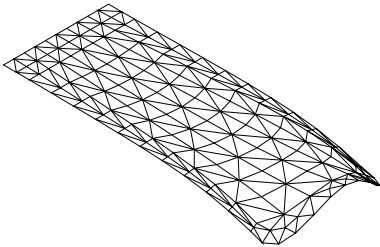
4794 Hz



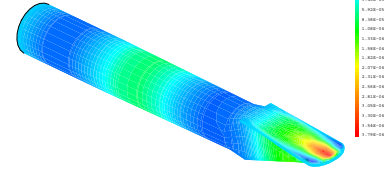
FC



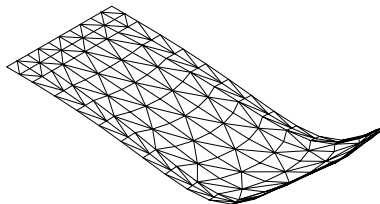
5897 Hz



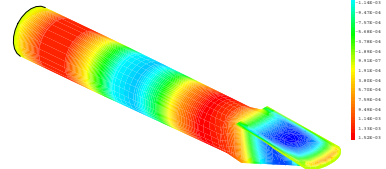
FT



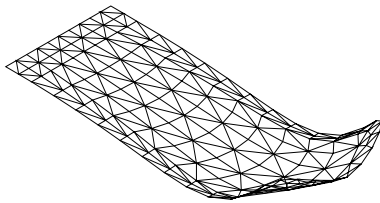
5983 Hz



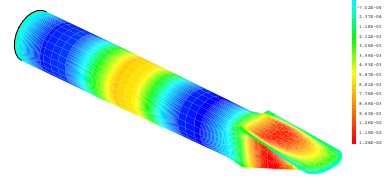
FL



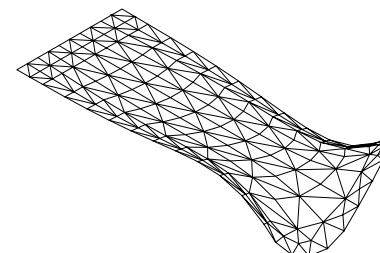
6491 Hz



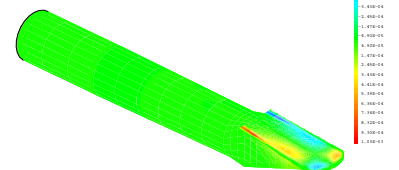
FC



7294 Hz



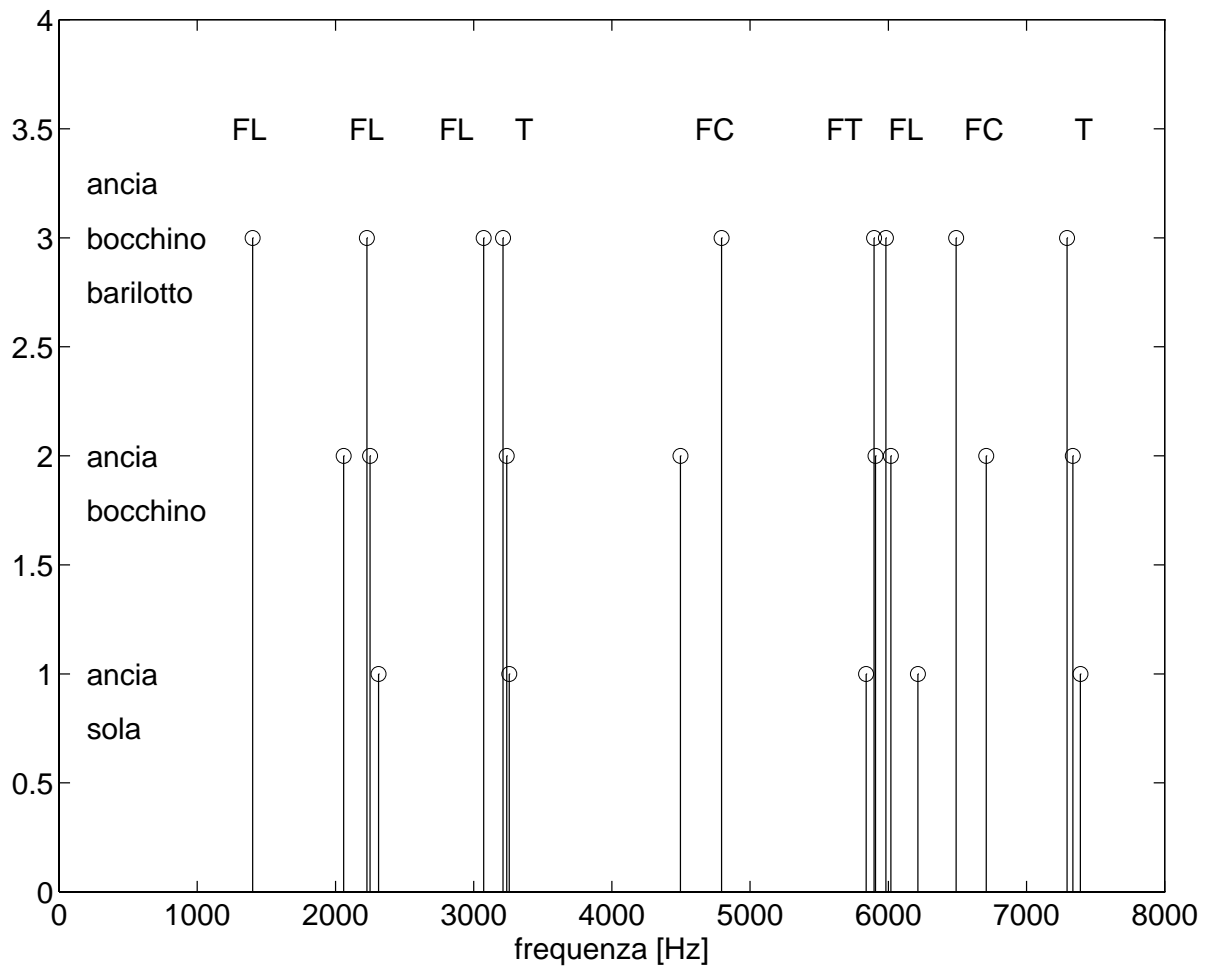
T



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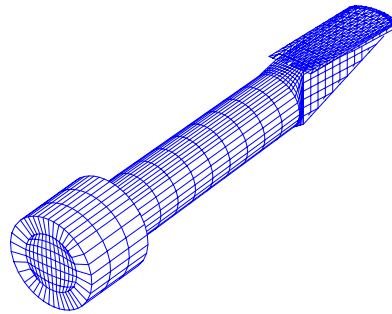
# COMPARISON

REED		REED + MOUTHPIECE + BARREL	
$f$ [Hz]	<i>eigenmode</i>	$f$ [Hz]	<i>eigenmode</i>
		1401	longitudinal flex
2312	longitudinal flex	2228	longitudinal flex
		3072	longitudinal flex
3257	torsion	3213	torsion
		4794	"composite" flex
5840	transversal flex	5897	transversal flex
		5983	longitudinal flex
6214	longitudinal flex	6491	"composite" flex
7389	torsion	7294	torsion



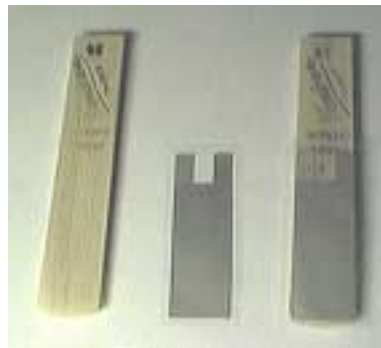
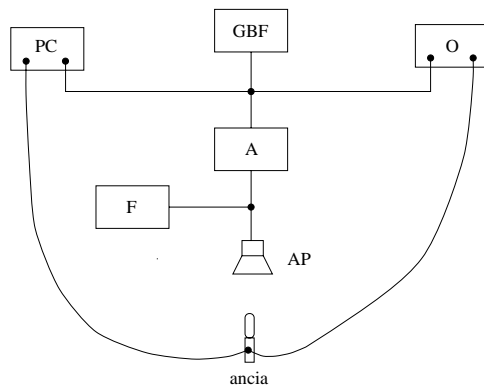
# EXPERIMENTAL VALIDATION

- **model:** reed + mouthpiece + barrel

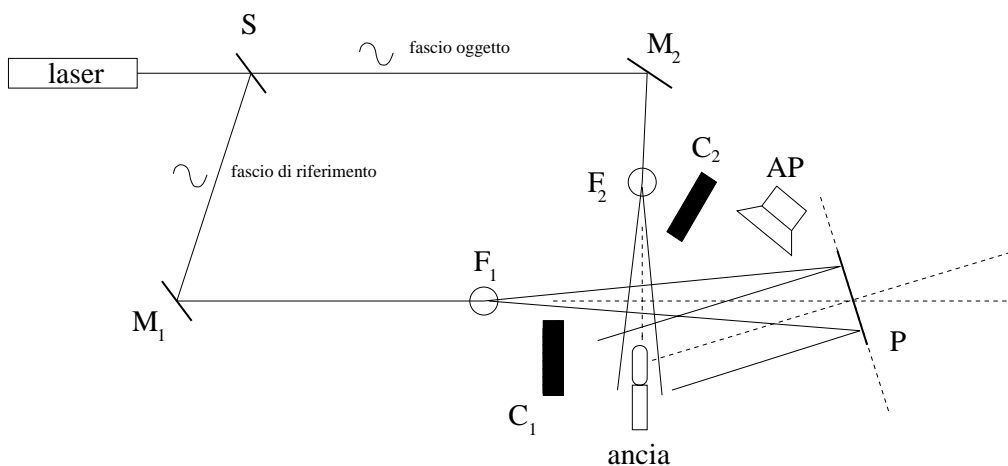


*limited reed oscillations → no contact on mouthpiece lay*

- **eigenfrequencies:** piezo-electric probe



- **eigenmodes:** interferometric holography



# COMPARISON

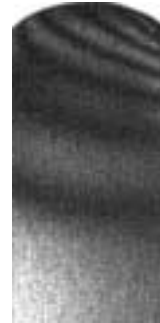
computation

measure

1190 Hz



FL



1100 Hz

2010 Hz



FL

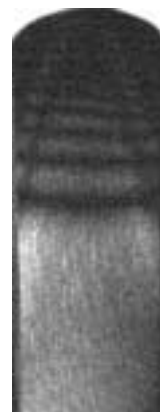


1600 Hz

2670 Hz



FC



2500 Hz

3700 Hz



T



3400 Hz

$$\lambda = 0.6328 \mu m$$

\*

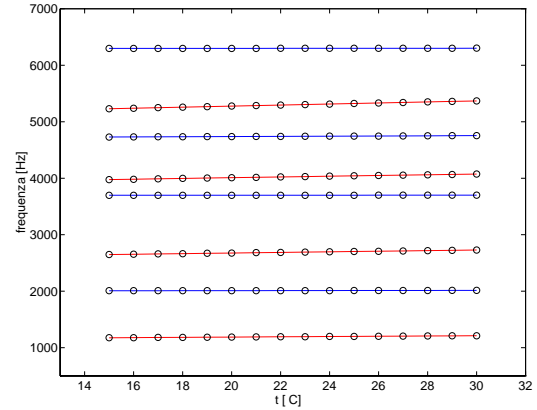
# SENSIBILITY ANALYSIS

- wet air physical properties:  $\rho, c = f(T, \varphi)$

mode	type	$\frac{Hz}{^\circ C}$
8	FT	0.2
7	FL	9.2
6	FC	1.6
5	FC	6.4
4	T	0.1
3	FC	5.3
2	FL	0.4
1	FL	2.4

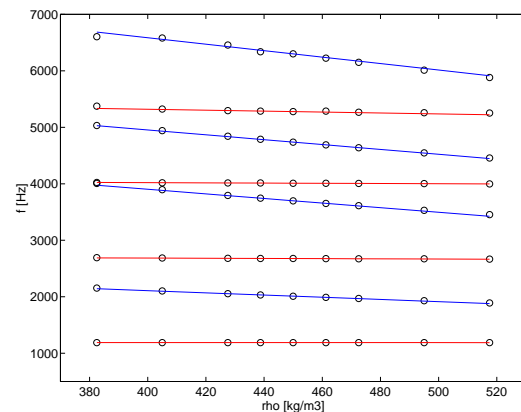
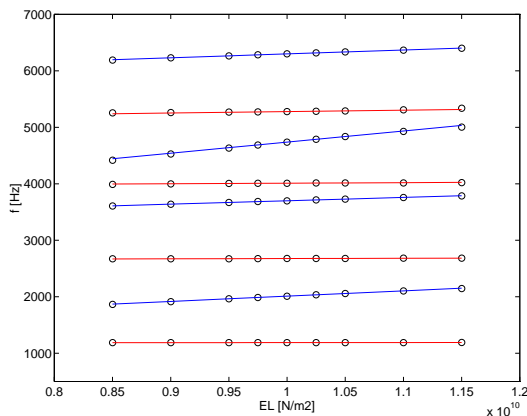
$$\varphi = 1$$

$p_{atm}$



- reed mechanical properties

$\Delta = 10\%$	$E_L$	$E_T$	$G_{LT}$	$\nu_{LT}$	$\rho$
mean values	10e4 MPa	400 MPa	1300 MPa	0.22	450 $\frac{kg}{m^3}$
FL	2.3%	<0.05%	<0.05%	<0.05%	-2.2%
FC	4.9%	<0.05%	<0.05%	<0.05%	-4.5%
FT	1.7%	0.9%	4.9%	<0.05%	-4.8%
T	1.4%	0.1%	3.1%	<0.05%	-6.4%



- cavity geometry  $\left\{ \begin{array}{l} 1 \text{ mm} \rightarrow 1\% \\ \alpha = 0^\circ \div 25^\circ \\ \text{"mouthpiece" mode } 2230 \text{ Hz} \end{array} \right.$



# REST OF THE INSTRUMENT

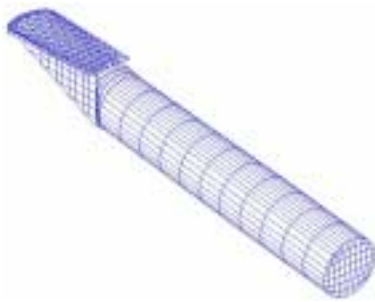
- **problem:** massive FEM model not efficient

$$\left\{ \begin{array}{l} \text{computationally expensive} \\ 1 \text{ note} \Leftrightarrow 1 \text{ model} \end{array} \right.$$

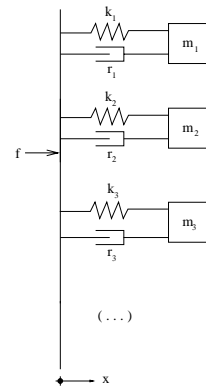
- **idea:** impedance simulation by dashpot system

$$Z_m(\omega) = \frac{f}{x} \quad Z_m(\omega) = S^2 \cdot Z_a(\omega) \quad Z_a(\omega) = \frac{p}{U}$$

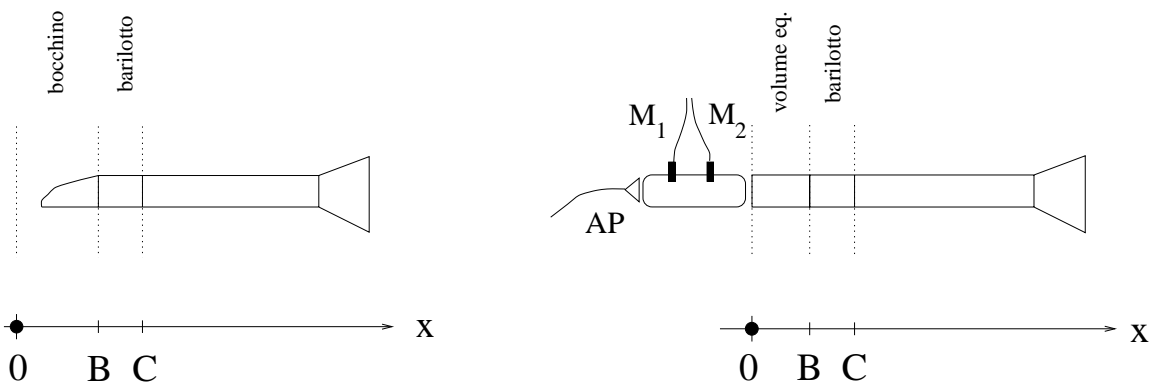
*useful adaptability*



1D hypothesis  
eigenmode analysis  
 $\forall$  note



- **measures:** [Vincent Gibiat (ESPCI) - 1998]

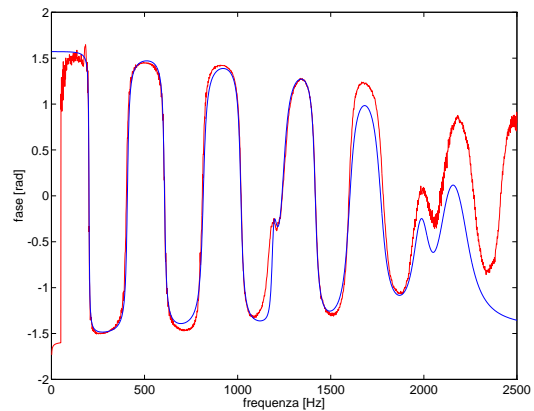
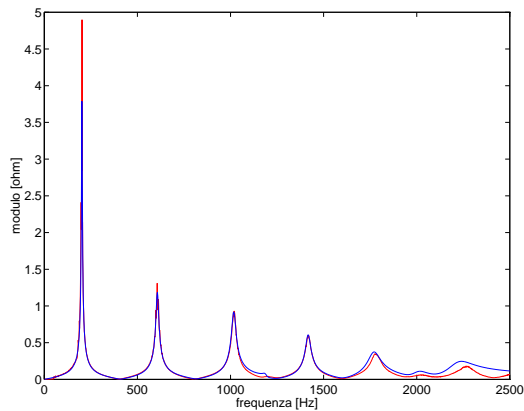


– mouthpiece volume correction

– analytic transformation:  $Z_a(O) \rightarrow Z_a(C)$

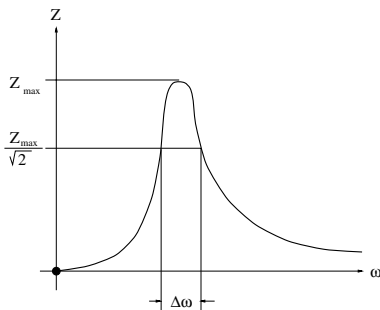
# IDENTIFICATION

Matlab

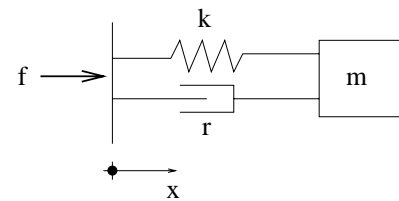


- preliminary data analysis  $\left\{ \begin{array}{l} \text{filtering (splines)} \\ \text{phase correction} \\ \text{incomplete parts} \end{array} \right.$

- single dashpot parameters: initial estimation



*eigenmode analysis*



- error function minimization

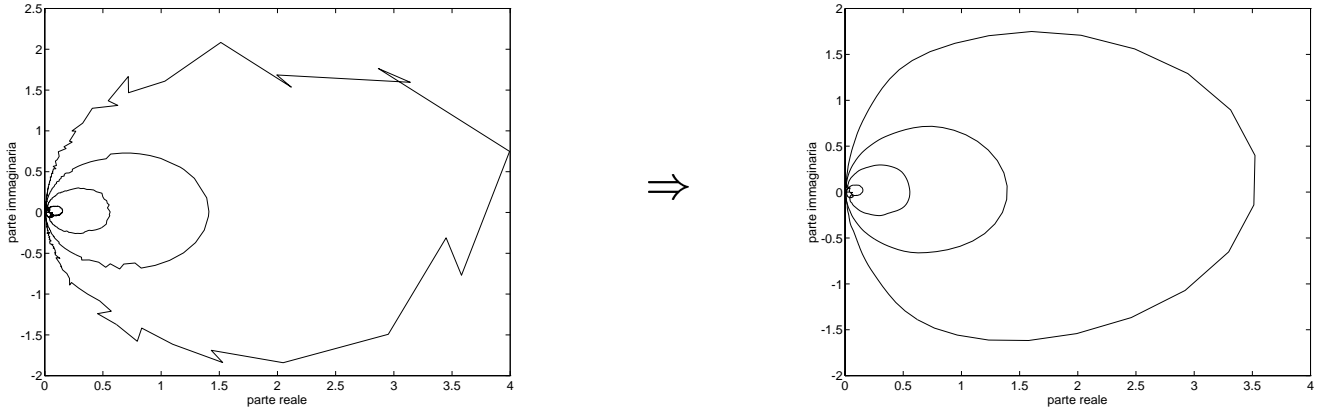
$$E = \int_{\Omega} |f_{\text{perimentale}}(\omega) - f_{\text{calcolata}}(\omega)|^2 d\omega$$

$$\begin{cases} E = \alpha E_{\text{modulo}} + \beta E_{\text{fase}} \\ \alpha + \beta = 1 \quad 0 \leq \alpha, \beta \leq 1 \end{cases}$$

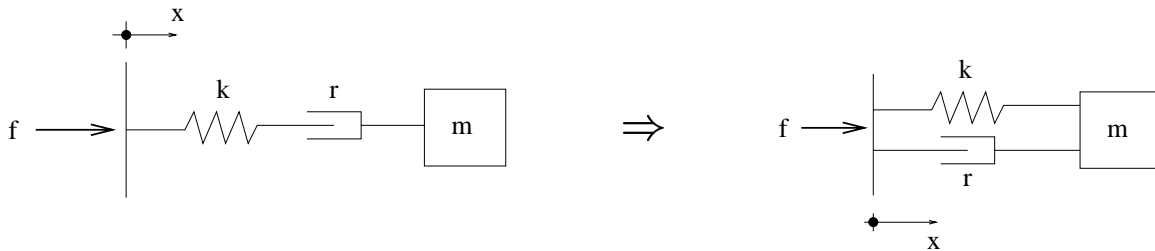
algorithm *Simplex Search - Nelder Mead*

# THOROUGH ANALYSIS

- **smoothing: cubic splines (Matlab)**

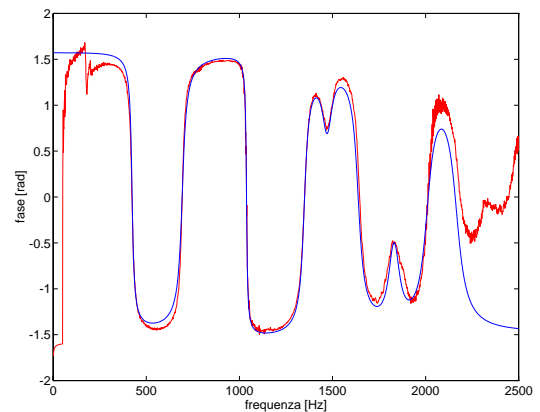
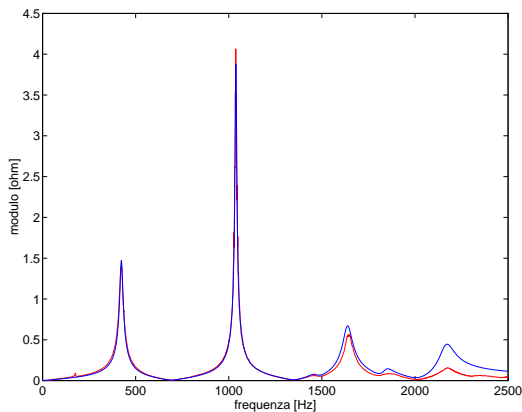


- **system transformation: chain  $\rightarrow$  comb**



- **progressive opt.**

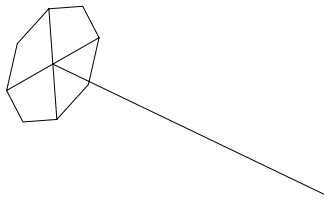
{ frequency domain  $\Delta f \uparrow$   
 module Vs phase  $\alpha \downarrow$



\*

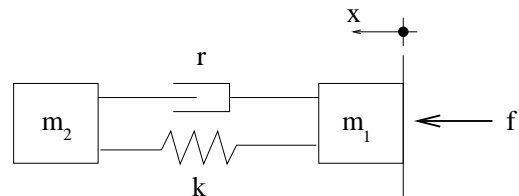
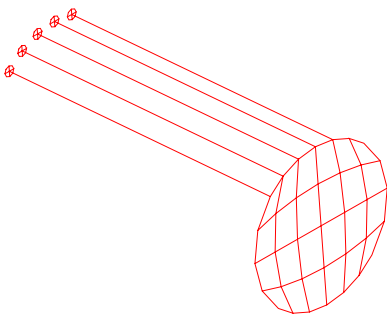
# NUMERIC IMPLEMENTATION

- **single dashpot**  $\left\{ \begin{array}{l} \text{mass} \quad \rightarrow \text{plate} \\ \text{stiffness} \quad \rightarrow \text{bar} \\ \text{damping} \quad \rightarrow \text{analytic} \end{array} \right.$



$$\frac{m_{plate}}{m_{bar}} \geq 10^3 \quad \Rightarrow \quad \Delta < 1\%$$

- **interface connection**  $\left\{ \begin{array}{l} \text{dashpot system} \\ \text{acoustical cavity} \end{array} \right.$

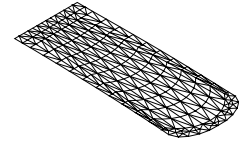
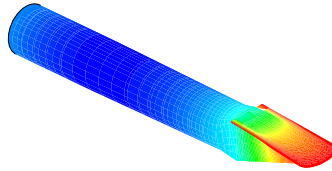
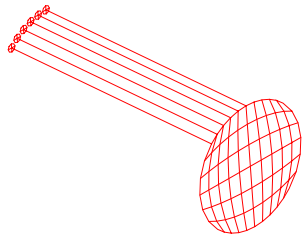


$$m_{connection} \leq 1.7 \cdot 10^{-8} \text{ kg} \quad \Rightarrow \quad \Delta < 1\%$$

# REED + INSTRUMENT

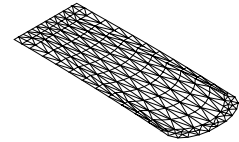
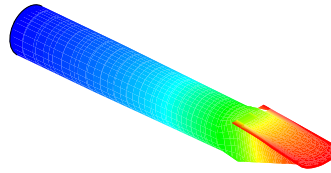
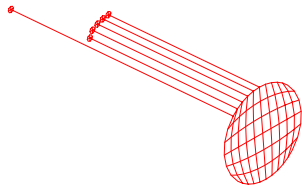
## real eigenmodes (1)

0 Hz



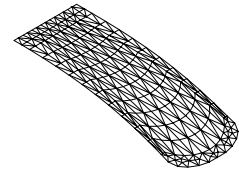
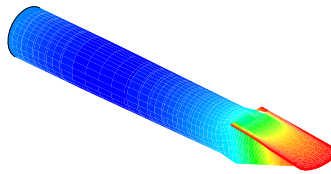
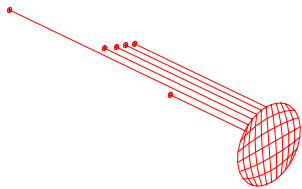
R

464 Hz



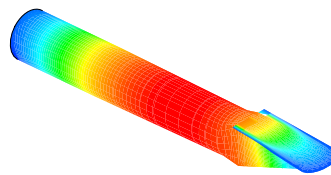
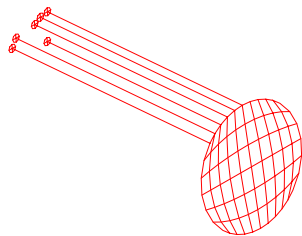
FL

873 Hz



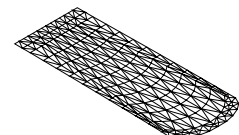
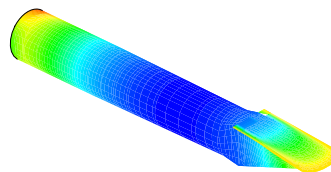
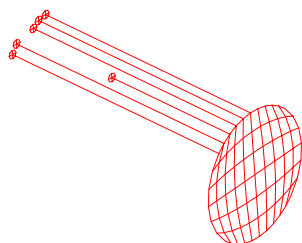
FL

1334 Hz



FL

1438 Hz

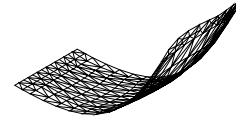
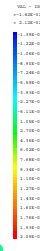
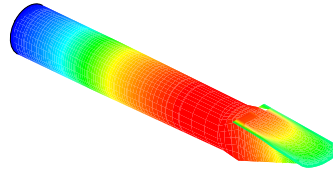
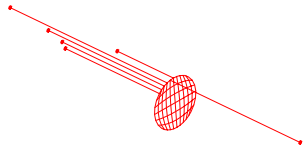


FL

# REED + INSTRUMENT

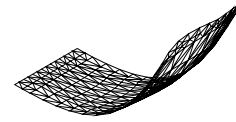
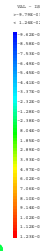
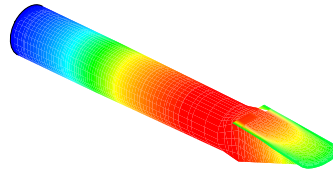
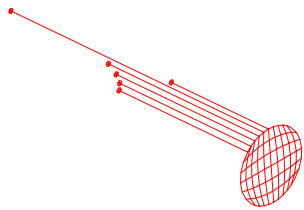
## real eigenmodes (2)

1767 Hz



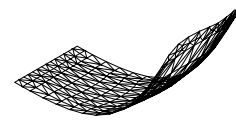
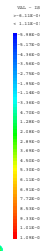
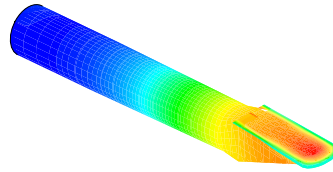
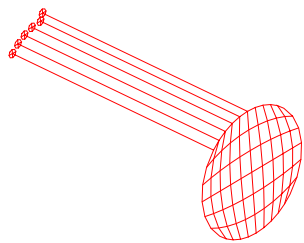
FL

1864 Hz



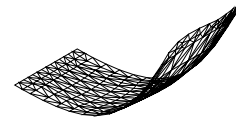
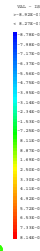
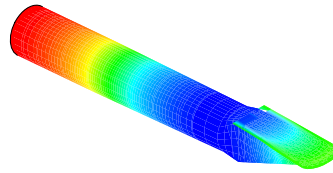
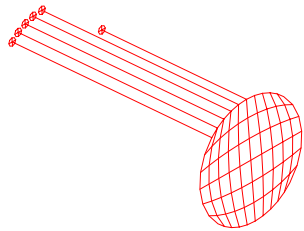
FL

1980 Hz



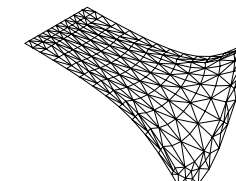
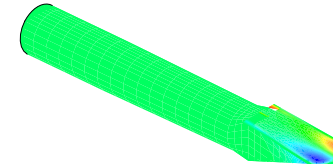
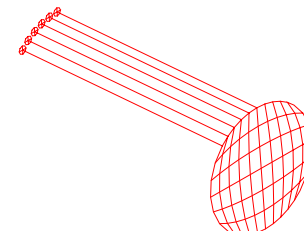
FL

2234 Hz



FL

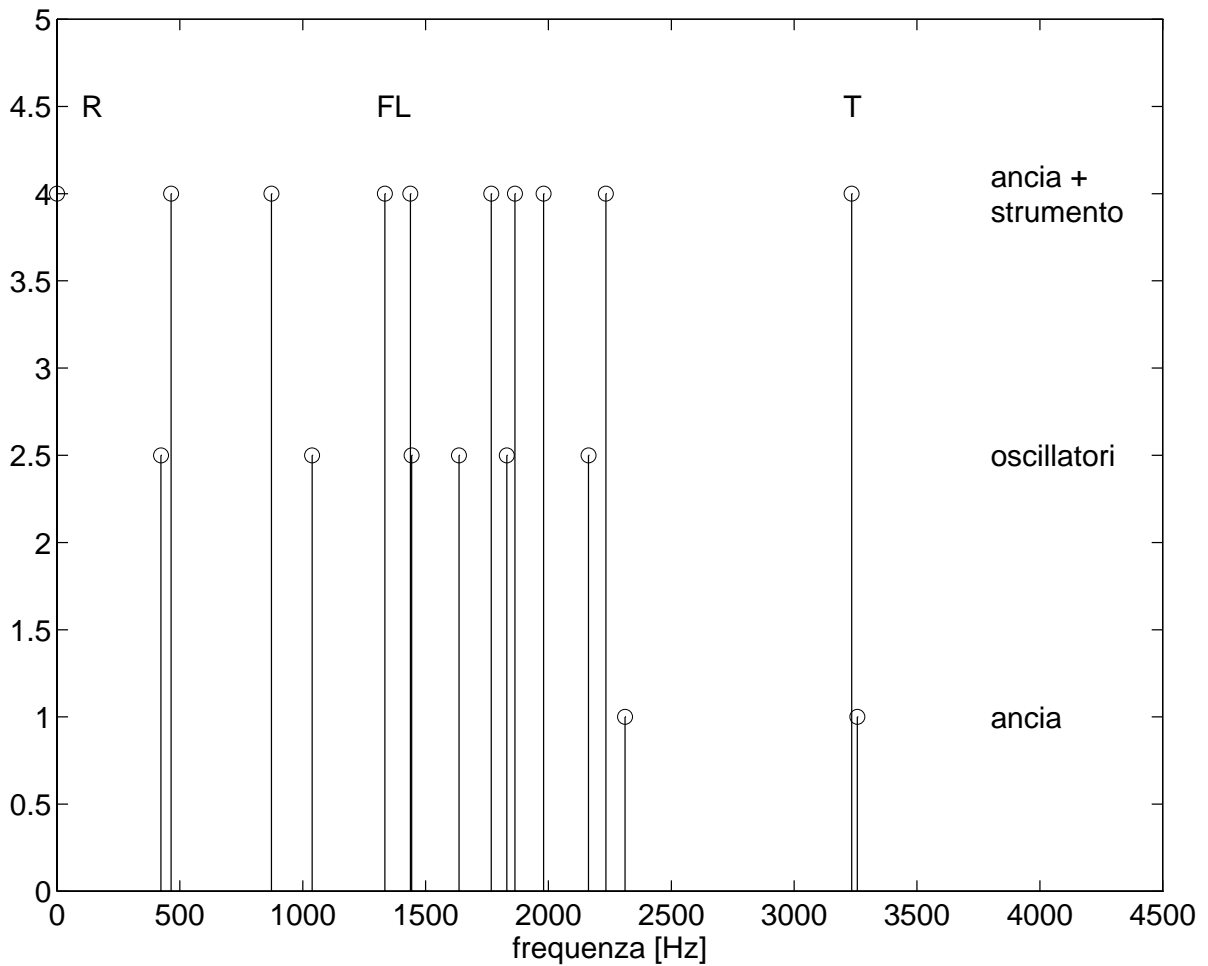
3234 Hz



T

# COMPARISON

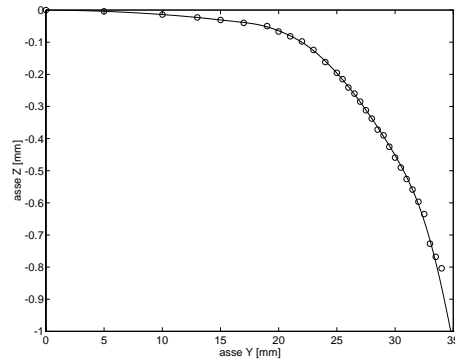
REED		REED + INSTRUMENT		DASHPOTS
$f$ [Hz]	eigenmode	$f$ [Hz]	eigenmode	$f$ [Hz]
		0	rigid	
		464	longitudinal flex	423
		873	longitudinal flex	1038
		1334	longitudinal flex	
		1438	longitudinal flex	1443
		1767	longitudinal flex	1636
		1864	longitudinal flex	1831
		1980	longitudinal flex	2163
2312	longitudinal flex	2234	longitudinal flex	
3257	torsion	3234	torsion	



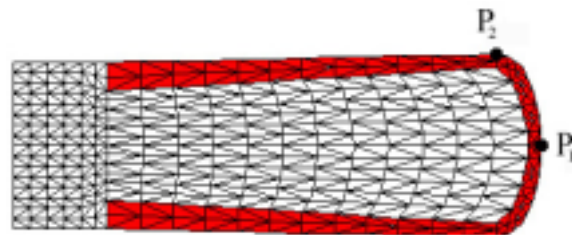
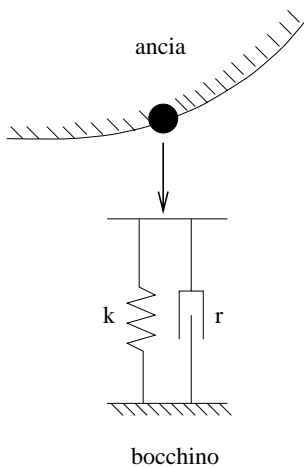
# BEATING REED (1)

## set-up

- geometry: CMM + Matlab



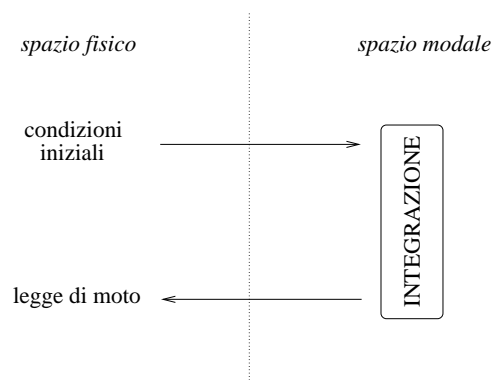
- contact law:  $f_{choc}(x, t) = k(u - u_0) + r \dot{u}$   
 $u(x, t) \geq u_0(x)$



- computation: eigenmode projection

$$u(x, t) \simeq \sum_{i=1}^N \alpha_i(t) w_i(x)$$

algorithm  
*Fu - de Vogelære*  
 4<sup>th</sup> ordre, explicit

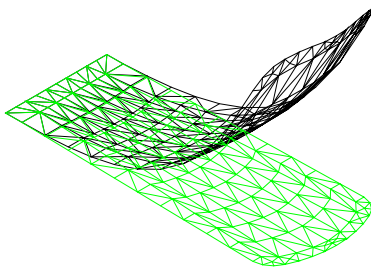
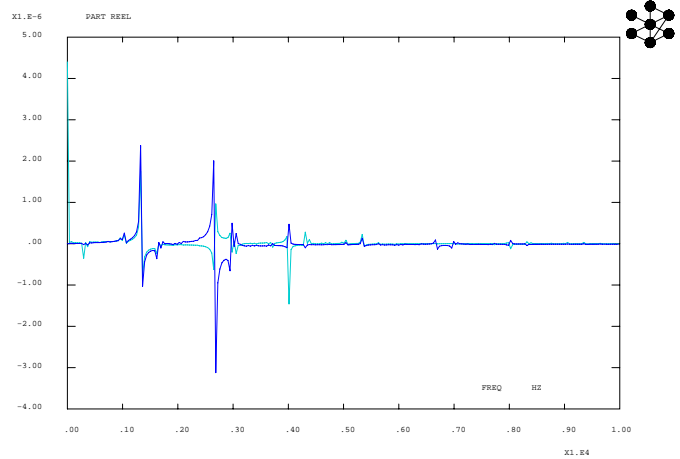
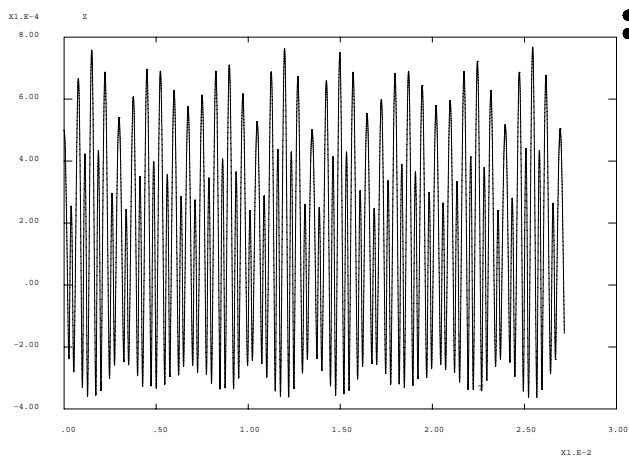
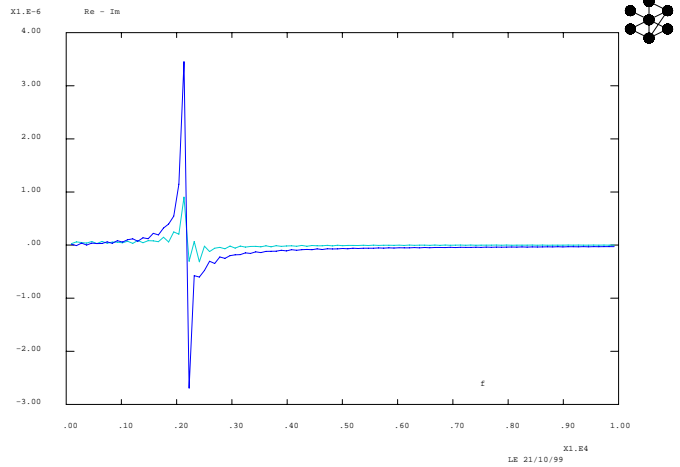
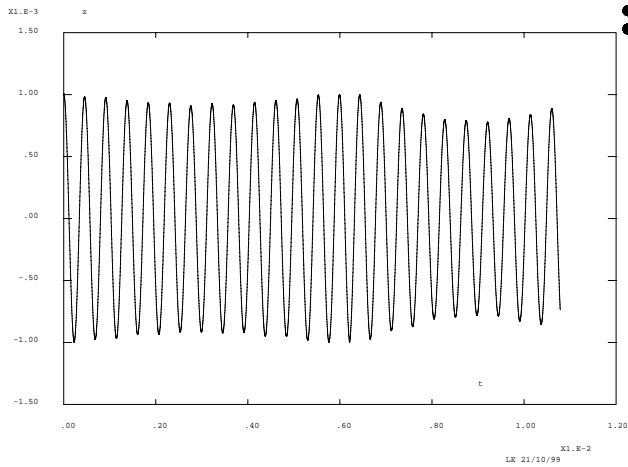




# BEATING REED (2)

## results

*displacement signals of a point of the reed*

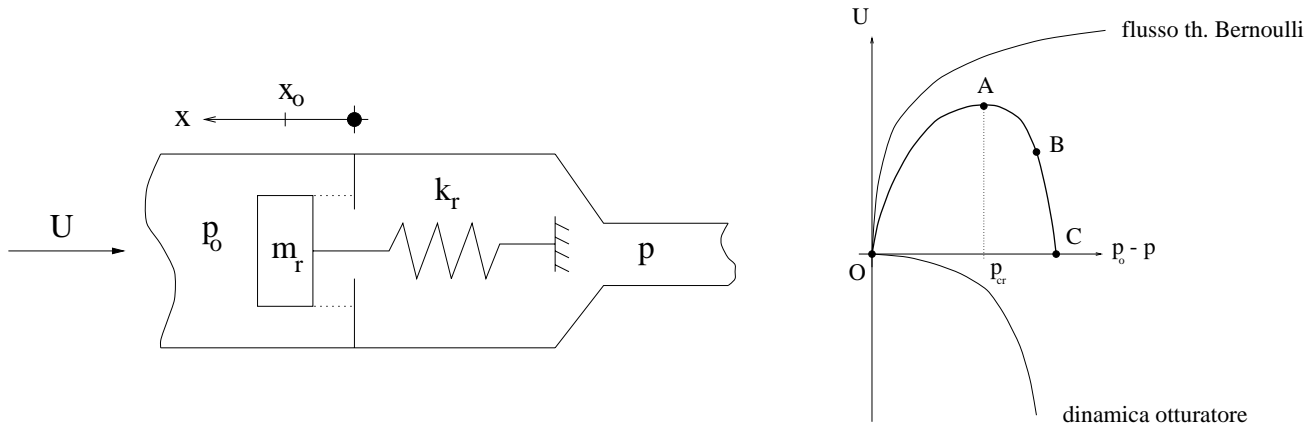


**effects** { deformation: asymmetric  $\rightarrow$  symmetric  
displacements  $\sim$  mouthpiece lay curvature  
"enriched" spectrum

# SOUND EMISSION (1)

## model

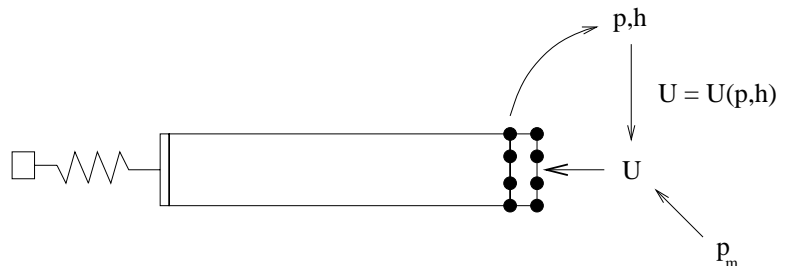
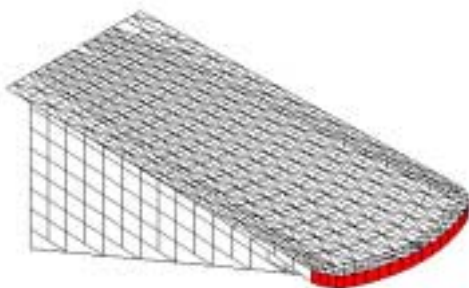
reed = pressure-controlled flux valve



acoustic admittance 
$$Y_r = -\frac{\partial U}{\partial p} = +\frac{\partial U}{\partial (p_0 - p)}$$

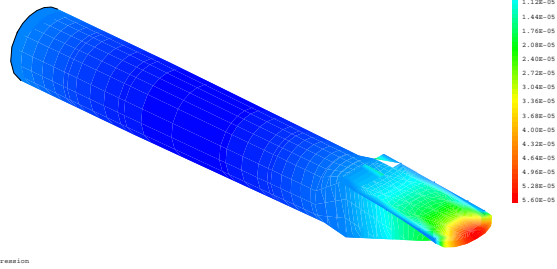
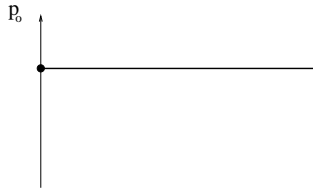
regime 
$$\left\{ \begin{array}{l} \text{reed dissipator:} \quad p_0 < p_{cr} \quad \rightarrow \quad Y_r > 0 \\ \text{reed generator:} \quad p_0 > p_{cr} \quad \rightarrow \quad Y_r < 0 \end{array} \right.$$

- *Bernoulli flux* → modified EF comportement law
- *cap dynamic* → numeric integration



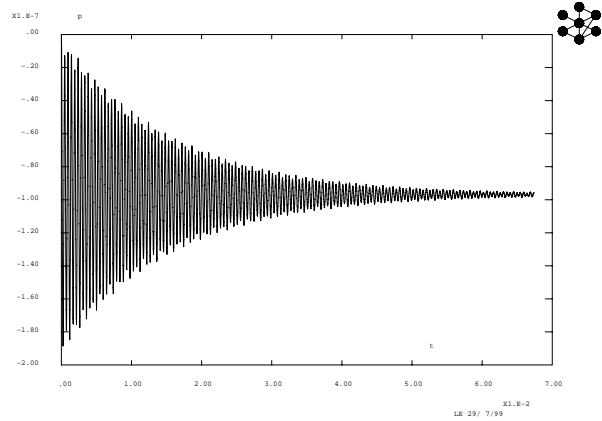
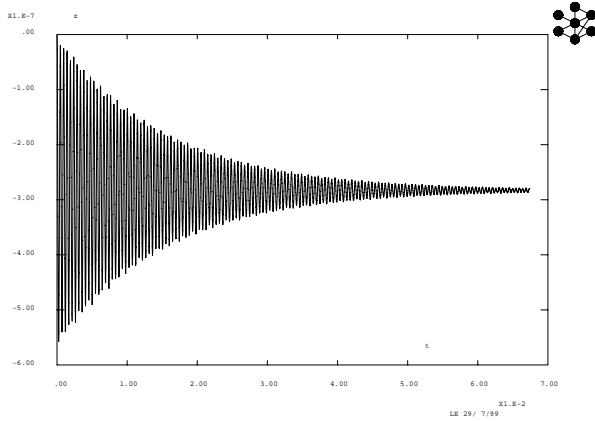
# SOUND EMISSION (2)

## results



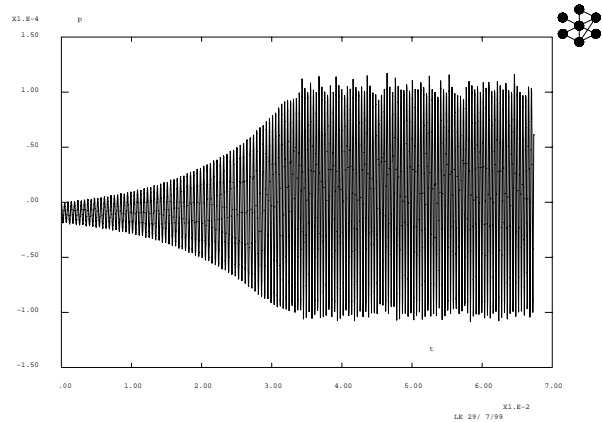
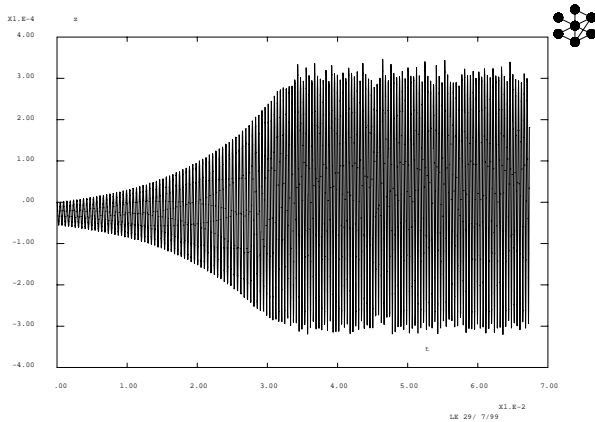
**dissipative regime:  $p_0 < p_{cr}$**

*displacement and acoustic pressure signals*



**generative regime:  $p_0 > p_{cr}$**

*displacement and acoustic pressure signals*



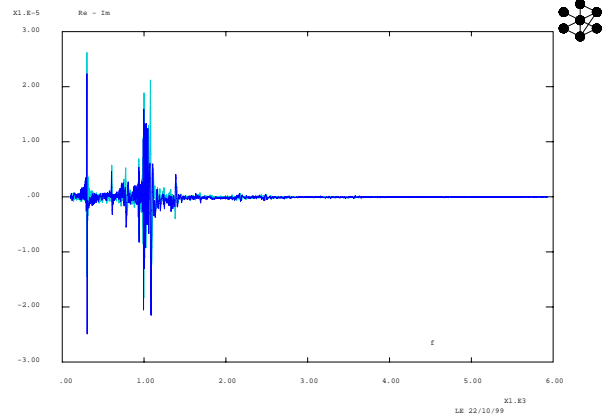
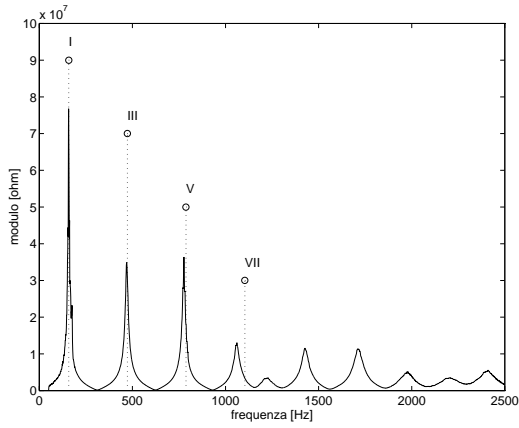
# SOUND EMISSION (3) results

**FA low**

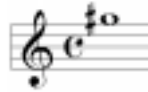


*measured acoustic impedance*

*computed sound spectrum*

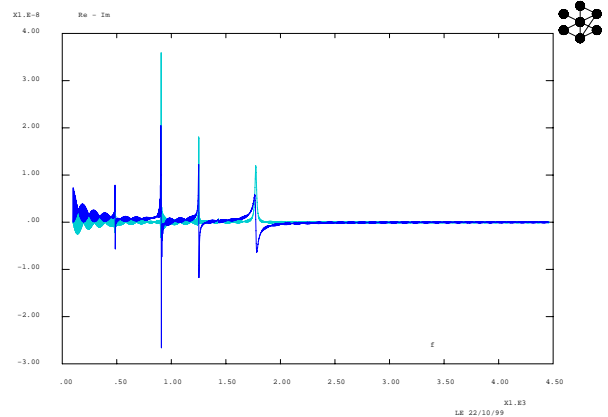
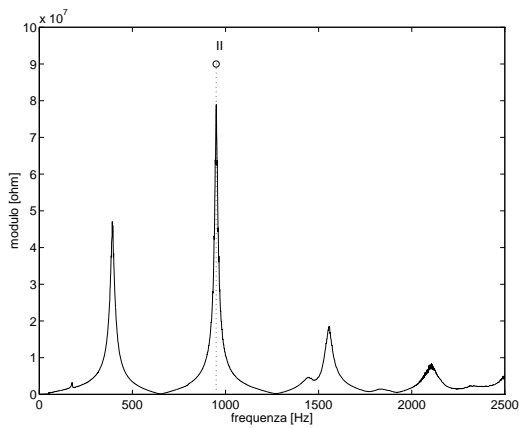


**SOL# high**



*measured acoustic impedance*

*computed sound spectrum*



# CONCLUSIONS & PROSPECTS

- **target:** numerical model  $\left\{ \begin{array}{l} \text{sound generation} \\ \text{beating reed} \\ \text{low frequencies} \end{array} \right.$
  
- **widening & development paths**
  - *spectrum*  $\left\{ \begin{array}{l} \text{frequencies } \uparrow \\ \text{“tone” of the clarinet} \end{array} \right.$
  
  - *player*  $\left\{ \begin{array}{l} \text{mechanic contact on the reed} \\ \text{mouth resonant cavity} \end{array} \right.$
  
  - *reed*  $\left\{ \begin{array}{l} \text{wet air } \rightarrow \text{ wet reed} \\ \text{local heterogeneity \& anisotropies} \end{array} \right.$
  
- **applications**  $\left\{ \begin{array}{l} \text{dynamic behavior and sound analysis} \\ \text{artificial reeds (composite)} \end{array} \right.$